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IDENTIFICATION NONLINEAR DYNAMIC SYSTEMS BASED ON VOLTERRA POLYNOMIALS WITH USING POLYHARMONIC TEST SIGNALS

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ABSTRACT. The method is proposed for constructing the Volterra approximation model of the nonlinear dynamical systems in the frequency domain using of the test polyharmonic signals of various amplitudes. The computing identification method is based on using of the regularized least squares method. The method improves an accuracy and stability of the identification procedure.

Introduction. A method of constructing approximation Volterra model of the nonlinear dynamical systems (NDS) is developing [1]. Method of the identification is based on the approximation of the response $y(t)$ system at an arbitrary deterministic signal $x(t)$ in the form of integral power of the polynomial Volterra N-th order (N – order approximation model):

$$\tilde{y}_N(t) = \sum_{n=1}^{N} \hat{y}_n(t) = \sum_{n=1}^{N} \underbrace{\int_0^\infty \dots \int_0^\infty}_{n \text{ times}} w_n(\tau_1,\dots,\tau_n) \prod_{i=1}^{n} x(t-\tau_i)\,d\tau_i. \quad (1)$$

The main part of work. The statement which proof is given in [1] is true.

Statement 1. Let the input test signals NDS are fed alternately $a_1x(t)$, $a_2x(t)$, ,…, $a_Lx(t)$; a_1, a_2, …, a_L – distinct real numbers satisfying the condition $|a_j| \le 1$ for $\forall j=1, 2, ..., L$; then

$$\tilde{y}_N[a_jx(t)] = \sum_{n=1}^{N} \hat{y}_n[a_jx(t)] = \sum_{n=1}^{N} a_j^n \underbrace{\int_0^\infty \dots \int_0^\infty}_{n \text{ times}} w_n(\tau_1,\dots,\tau_n) \prod_{i=1}^{n} x(t-\tau_i)\,d\tau_i = \sum_{n=1}^{N} a_j^n \hat{y}_n(t). \quad (2)$$

The partial components in the approximation model $\hat{y}_n(t)$ are found using the least square method. This makes it possible to obtain such evaluation in which the sum of squared deviations of responses identified the nonlinear dynamical system $y[a_jx(t)]$ on the model $\hat{y}_N[a_jx(t)]$ response is minimal, i.e., NDS provides a minimum criterion

$$J_N = \sum_{j=1}^{L} \left(y[a_jx(t)] - \tilde{y}_N[a_jx(t)] \right)^2 = \sum_{j=1}^{L} \left(y_j(t) - \sum_{n=1}^{N} a_j^n \hat{y}_n(t) \right)^2 \to \min, \quad (3)$$

where $y_j(t) = y[a_jx(t)]$. Minimization of the criterion (3) is reduced to solving the system of normal equations of Gauss, which in vector-matrix form can be written as

$$\mathrm{A}'\mathrm{A}\,\hat{\mathrm{y}} = \mathrm{A}'\,\bar{\mathrm{y}}, \quad (4)$$

where

$$\mathrm{A} = \begin{bmatrix} a_1 & a_1^2 & \cdots & a_1^N \\ a_2 & a_2^2 & \cdots & a_2^N \\ \cdots & \cdots & \cdots & \cdots \\ a_L & a_L^2 & \cdots & a_L^N \end{bmatrix}, \quad \bar{\mathrm{y}} = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \cdots \\ y_L(t) \end{bmatrix}, \quad \hat{\mathrm{y}} = \begin{bmatrix} \hat{y}_1(t) \\ \hat{y}_2(t) \\ \cdots \\ \hat{y}_N(t) \end{bmatrix}.$$

The Tikhonov regularization method based on the variational method of constructing a regularizing operator is used to obtain the error-resistant SLAE (4) solution [2]. This method is to determine an approximative solution vector that minimizes a certain smooth functionality. The only vector satisfying the minimum condition of the smoothing functional is determined on the basis of the SLAE solution

$$(\mathrm{A}'\mathrm{A} + \alpha \mathrm{I})\hat{\mathrm{y}}_\alpha = \mathrm{A}'\,\bar{\mathrm{y}}, \quad (5)$$

where A′ – transposed matrix; I – the identity matrix; and α is a parameter of regularization.

When implementing this algorithm, the regularization parameter α is chosen quite small (from the analysis of the available information about the error of the input data and the error of calculations). The value of the regularization parameter α is determined by matching, that is, multiple calculations $\hat{y}_\alpha$, for different values α.

For identification in the frequency domain the test polyharmonic signals are used. We prove:

Statement 2. If test polyharmonic signal is used in form

$$x(t) = A\sum_{k=1}^{n} \cos \omega_k t = \frac{A}{2}\sum_{k=1}^{n}\left(e^{j\omega_k t} + e^{-j\omega_k t}\right), \tag{6}$$

then the n-th partial component of the response of test system can be written in the form:

$$y_n(t) = \frac{A^n}{2^{n-1}}\sum_{m=0}^{E(n/2)} C_n^m \sum_{k_1=1}^{n}\ldots\sum_{k_n=1}^{n}\left|W_n\left(-j\omega_{k_1},\ldots,-j\omega_{k_m}, j\omega_{k_{m+1}},\ldots, j\omega_{k_n}\right)\right| \times$$
$$\times \cos\left(\left(-\sum_{l=0}^{m}\omega_{k_l} + \sum_{l=m+1}^{n}\omega_{k_l}\right)t + \arg W_n\left(-j\omega_{k_1},\ldots,-j\omega_{k_m}, j\omega_{k_{m+1}},\ldots, j\omega_{k_n}\right)\right), \tag{7}$$

where E() – function used to obtain the of integer part of the value. The partial components for n=1, 2 and 3 are the form, respectively

$$y_1(t) = A\left|W_1(j\omega)\right|\cos(\omega t + \arg W_1(j\omega)) \tag{8}$$

$$y_2(t) = \frac{A^2}{2}\sum_{k_1,k_2=1}^{2}\left|W_2(j\omega_{k_1}, j\omega_{k_2})\right|\cos((\omega_{k_1}+\omega_{k_2})t + \arg W_2(j\omega_{k_1},\omega_{k_2})) +$$
$$+ A^2\sum_{k_1,k_2=1}^{2}\left|W_2(-j\omega_{k_1}, j\omega_{k_2})\right|\cos((-\omega_{k_1}+\omega_{k_2})t + \arg W_2(-j\omega_{k_1},\omega_{k_2})) \tag{9}$$

$$y_3(t) = \frac{A^3}{4}\sum_{k_1,k_2,k_3=1}^{3}\left|W_3(j\omega_{k_1}, j\omega_{k_2}, j\omega_{k_3})\right|\cos((\omega_{k_1}+\omega_{k_2}+\omega_{k_3})t + \arg W_3(j\omega_{k_1}, j\omega_{k_2}, j\omega_{k_3})) +$$
$$+\frac{3A^3}{4}\sum_{k_1,k_2,k_3=1}^{3}\left|W_3(-j\omega_{k_1}, j\omega_{k_2}, j\omega_{k_3})\right|\cos((-\omega_{k_1}+\omega_{k_2}+\omega_{k_3})t + \arg W_3(-j\omega_{k_1}, j\omega_{k_2}, j\omega_{k_3})) +$$
$$+\frac{3A^3}{4}\sum_{k_1,k_2,k_3=1}^{3}\left|W_3(-j\omega_{k_1}, -j\omega_{k_2}, j\omega_{k_3})\right|\cos((-\omega_{k_1}-\omega_{k_2}+\omega_{k_3})t + \arg W_3(-j\omega_{k_1}, -j\omega_{k_2}, j\omega_{k_3})) \tag{10}$$

The component with frequency $\omega_1+\ldots+\omega_n$ is extracted from the response to test signal (7):

$$A^n \left|W_n(j\omega_1,\ldots, j\omega_n)\right| \cos[(\omega_1+\ldots+\omega_n)t + \arg W_n(j\omega_1,\ldots, j\omega_n)]. \tag{11}$$

Certain limitations should be imposed while choosing of frequences polyharmonic test signals in the process determine multidimensional AFC and PFC [3].

Conclusion. The method for building the Volterra approximation model of the nonlinear dynamical systems with using of the test polyharmonic signals of various amplitudes is proposed. The computing identification method is based on the use of the regularized least squares method. The method improves an accuracy and stability of the identification procedure in the form of multidimensional frequency characteristics of amplitude and phase.

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