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Analysis of quasi-periodic space-time non-separable processes to support decision-making in medical monitoring systems

Olena D. Franzheva
ORCID: https://orcid.org/0000-0003-2639-4012; franzheva.helen@op.edu.ua
Odessa National Polytechnic University, 1, Shevchenko Ave. Odessa, 65044, Ukraine

ABSTRACT

In many decision support systems there are processed chaotic spatial-time processes which are non-separable and quasi-periodic. Some examples of such systems are epidemic spreading, population development, fire spreading, radio wave signals, image processing, information encryption, radio vision, etc. Processes in these systems have periodic character, e.g. seasonal fluctuations (epidemic spreading, population development), harmonic fluctuations (pattern recognition, image processing), etc. In simulation block the existing systems use separable process models which are presented as multiplication of spatial and temporal parts and are linearized. This significantly reduces the quality of spatial-time non-separable processes. The quality model building of chaotic spatial-time non-separable process which is processed by decision support system is necessary for getting of learning set. It is really complicated especially if the random process is formed. The implementation ensemble of chaotic spatial-time non-separable process requires high costs what causes reduction of the system efficiency. Moreover, in many cases the implementation ensemble of spatial-time processes is impossible to get. In this work the mathematical model of a quasi-periodic spatial-time non-separable process has been developed. Based on it the formation method of this process has been developed and investigated. The epidemic spreading processed was presented as an example.

Keywords: Computer systems; decision making; quasi-periodic processes; spatial-time models; non-separable processes; cellular automata; dynamical systems; epidemic spreading

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INTRODUCTION

At the present stage there are widely used DSS in practice where it is necessary to analyze and simulate quasi-periodic spatial-time non-separable processes gone through processing. Harmonic and seasonal fluctuations and other cycle characteristics are specific for these processes. Such processes appear during epidemic spreading, population development, fire spreading, pattern recognition, image processing, information encryption, radio vision, etc.

At this moment developed models of spatial-time processes of the epidemic spreading and based on these models created decision support systems use dynamical compartment population models which divide whole population count on categories of disease stage. The most popular compartment model is SIR model (Susceptible, Infected and Recovered). The first important developments of compartment models were presented by Ross and Hudson in [1, 2], [3]. This model was improved in SIR epidemic spreading model by Kermack and McKendrick [4] and Kendall [5]. These systems have a form of differential equations and it makes difficult to build ones in computer simulation tasks. In work of White, Rey and Sa´nchez [6] the discrete SIR model was presented. It uses cellular automata that allow accounting spatial structure and solving the problem of computer simulation and information processing. Thus, existing separable models that are linearized and don't account quasi-periodic character of spatial-time non-separable processes.

It leads to DSS developers to have problems with promptness of learning set accumulation because of restrictions of existing. It significantly affects the reliability of the decisions made [7, 8], [9, 10], [11].

Only by correctly chosen model it is possible to predicate dynamic of chaotic process, analyze result and based on it stabilize the process.

Thus, increasing of these processes simulation quality is actual and important scientific and technical task.

THE PURPOSE OF THE PAPER

The purpose of the paper is to show how to use the quasi-periodic spatial-time non-separable processes simulation and formation methods development in decision support systems in order to increase simulation quality and data processing promptness.

FORMULATION OF THE PROBLEM

DSS are widely used in different human activity areas and usually include simulation, formation and analysis in systems with different nature (physical, biological, social, technical systems etc.). DSS utilization gives a possibility to decision maker (DM) to make an optimal by definite characteristics strategy choice.

It would lead to specific goals:

- The increasing of the business processes control efficiency;
- The quality control in the engineering;
- The risks evaluation in financial operations;
- The increasing of disease diagnostic accuracy in medicine;
- The monitoring and prediction of processes development in ecology (population development, fire spreading, environmental pollution, epidemic spreading etc.).

The common schema of the decision support system is presented on Fig. 1.

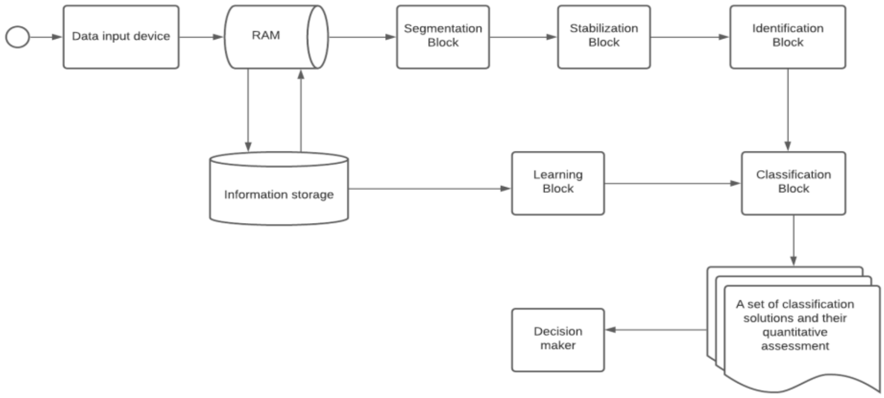

Fig.1. **The common schema of the decision support system**
***Source:* compiled by the author**

The learning block provides a formation of the representative learning set. It has the similar structure as DSS itself, but also teacher indication is included. (Fig. 2).

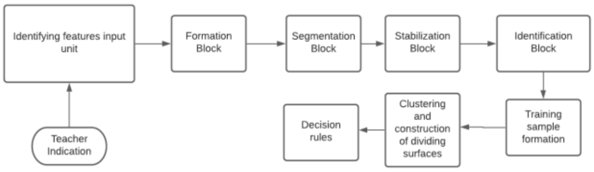

Fig.2. **DSM learning block schema**
***Source:* compiled by the author**

The spatial-time non-separable process formation block is based on a mathematical model. The simulation quality provided by this block much more defines efficiency of these DSS (reliability of decisions).

THE SPATIAL-TIME NON-SEPARABLE PROCESS SIMULATION METHOD

The quality model building of chaotic spatial-time non-separable process, which is processed by DSS, is necessary for getting of learning set because it is really complicated, especially if random process is formed. The implementation ensemble of chaotic spatial-time non-separable process requires high costs, what causes decreasing of the system promptness. Moreover, in many cases the implementation ensemble of spatial-time processes is impossible to get.

Existing models of spatial-time processes usually are separable:

$$s(x,y,t) = f_1(x,y) \cdot f_2(t)\,, \qquad (1)$$

where: x, y – spatial coordinates; t – time.

In nature spatial-time processes are characterized by cyclicality (Fig. 3).

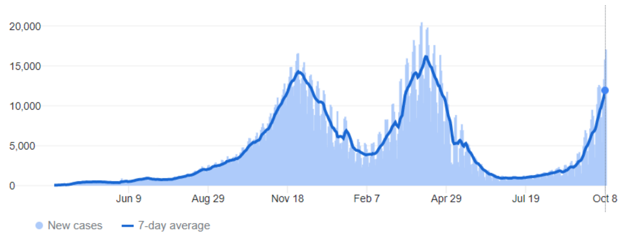

Fig.3. **COVID-19 disease statistics in Ukraine in 2021**
***Source:* compiled by the author**

In this case the time function in (1) has the following form:

$$f(t) = f(t + kT_\xi), \quad k = 1,2,\ldots,n\,, \qquad (2)$$

where period T_ξ – random variable.

In the number of applied tasks (e.g. process of epidemic spreading simulation) the separation of spatial and temporal parts in simulation of such

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processes by scheme (1) is fundamentally impossible. That is why it is necessary to build a non-separable model which would account quasi-periodic character of the process.

Let us consider the mathematical SIR model of a quasi-periodic spatial-time non-separable process of epidemic spreading:

$$\begin{cases} \dfrac{dS}{dt} = -\beta \dfrac{SI}{N} \\ \dfrac{dI}{dt} = \beta \dfrac{SI}{N} - \gamma I \\ \dfrac{dR}{dt} = \gamma I \end{cases}, \qquad (3)$$

where: β – infection speed; γ – recovering speed; $S(t)$ – susceptible count; $I(t)$ – infected count, $R(t)$ – recovered count.

This model doesn't have separation of spatial and temporal parts of process and its linearization at the next step. The model (3) can be used for formation of spatial-time non-separable process and getting of a learning set.

In this work we haven't made a verification of the mathematical model, however, the quasi-periodic spatial-time non-separable process is formed based on it and the model quality experiment has been conducted.

FORMATION METHOD

Formation of chaotic spatial-time non-separable processes by construction of differential equations, as it is done in (3), is not effective enough, especially for computer simulation. It is easier to use cellular automata as formation method of spatial-time non-separable process to avoid a spatial and temporal parts multiplication approach and save its non-separability.

This method is appropriate to form processes in complex systems which consist of simple connected objects. It allows getting and researching complex system behavior all in all. However, classical cellular automata (CA) work is based on processing of logical rules. And increasing of rules and neighbors count leads to decreasing of system promptness.

In papers [12, 13] CA and its analytical representation is reviewed. Let us provide the algorithm of process formation method.

1. It is necessary to define the way of neighbors accounting. At the present stage there are two basic classifications of neighbors accounting and their multiple modifications: Von Neumann neighborhood – aggregation of neighbors which have common edges with current cell; Moore neighborhood – aggregation of neighbors which have common peaks with current cell. This method uses Moore neighborhood. To achieve that let us present a cell and its neighbors not in form of a lattice but as a strip, where $x_9(n)$ – cell state at the moment of time n, $x_j(n),\ j=\overline{1,8}$ – its neighbors states (Fig.4).

$x_1(n)$	$x_2(n)$	$x_3(n)$	$x_4(n)$	$x_5(n)$	$x_6(n)$	$x_7(n)$	$x_8(n)$	$x_9(n)$

Fig.4. **Moore neighborhood**
Source: **compiled by the author**

2. The next step is to define corresponding weight coefficients [13] and represent them as matrix D:

$$\begin{pmatrix} \delta_9 & 0 & \cdots & 0 & \delta_1 & \delta_2 & \delta_3 & \delta_4 & \delta_5 & \delta_6 & \delta_7 & \delta_8 \\ \delta_8 & \delta_9 & 0 & \cdots & 0 & \delta_1 & \delta_2 & \delta_3 & \delta_4 & \delta_5 & \delta_6 & \delta_7 \\ & & & & & & \cdots & & & & & \\ \delta_1 & \delta_2 & \delta_3 & \delta_4 & \delta_5 & \delta_6 & \delta_7 & \delta_8 & \delta_9 & 0 & \cdots & 0 \end{pmatrix}.$$

In simulation block of DSS the set of these weight coefficients defines the class of a random process implementation. Thus, the neighbors' impact on current cell degree is defined.

3. Further, it is necessary to build the Diffusion equation:

$$\begin{cases} y_i = \sum_{s=1}^{r} \delta_s x_{i+s}(n) \\ i = \overline{1,K} \end{cases}, \qquad (4)$$

where $r = 9$ – accounted cells count (cell itself and its neighbors).

Or if represent (4) in matrix form:

$$\mathrm{Y}_n = D\mathrm{X}_n. \qquad (5)$$

4. Choose and build the Reaction equation in correspondence to spatial-time process model, which is represented in matrix form as:

$$\mathrm{X}_{n+1} = \Phi(D\mathrm{X}_n), \qquad (6)$$

where: $X_n = \begin{pmatrix} x_1(n) \\ \vdots \\ x_K(n) \end{pmatrix}$ – vector which provides cells state and has dimension K; K – total number of cells in CA.

Generally speaking, elements $x_j(n)$ could be as numbers as vectors; $\Phi = \begin{pmatrix} \varphi_1 \\ \vdots \\ \varphi_K \end{pmatrix}$ – vector function, that describes cells states change.

Since present epidemical models are compartment, to form the epidemic spreading process in this work the cell state will be considered as vector that consists of three components $x_n = \begin{pmatrix} S \\ I \\ R \end{pmatrix}$.

As at the model (3), here S – susceptible count; I – infected count; R – recovered count.

In paper [14] φ_i, $i = \overline{1, K}$ epidemic spreading process formation method is:

$$\begin{cases} I(n+1) = (1-\varepsilon)I(n) + \upsilon S(n) \times \\ \qquad \times \left(\alpha I(n) + (1-\alpha) \sum_{a,b=-1}^{1} \delta_{ab} I_{i+a,j+b}(n) \right) \\ S(n+1) = S(n) + \mu R(n) - \upsilon S(n) \times \\ \qquad \times \left(\alpha I(n) + (1-\alpha) \sum_{a,b=-1}^{1} \delta_{ab} I_{i+a,j+b}(n) \right) \\ R(n+1) = (1-\mu)R(n) + \varepsilon I(n) \end{cases} \quad (7)$$

where: n – iteration; ε is chosen as inverse variable of disease duration; μ – inverse variable of immunity keeping duration; υ – coefficient of probability of infestation during contact with infected unit; α – level of society mobility (means the coefficient of infected units impact on susceptible ones in dependence of contact frequency).

The value $\sum_{a,b=-1}^{1} \delta_{ab} I_{i+a,j+b}(n)$ – neighbors' infected part impact on cell current state; δ_{ab} – weight coefficients defined in step 2.

In epidemic spreading process formation method proposed in this work by (4-7) for stabilization block we can apply control system which uses generalized semi linear delayed feedback control developed in [15] and improved predicative control developed in [16].

SIMULATION QUALITY EVALUATION

It is necessary to build process implementation ensemble for DSS learning set to classify four danger COVID-19 disease zones: green, yellow, orange, red. Analytical presented CA is suggested to be used in these random process implementations formations. For formation of process specified on exact COVID-19 disease zone weight coefficients provided in special way are used.

Let us compare suggested in this paper model, linearized and separable models used in existing DSS with real process in order to show that mutual usage of CA, which allow describing adequately spatial part of spatial-time non-separable process and state space in dynamical systems' theory, which allows describing state value changes, shows better results on simulation quality evaluation.

Then evaluate normalized root mean square error for each of models:

$$\varepsilon = \frac{\sqrt{\frac{1}{n} \sum_{i=1}^{n} \left(\hat{X}_i - X_i \right)^2}}{\varepsilon_{\max}} \quad (8)$$

where: $\hat{X}_i$ – real process data; X_i – simulated process data; $\varepsilon_{\max}$ – maximum value of a root mean square error; n – set elements number.

COVID-19 epidemic spreading process is considered as real spatial-time non-separable process. Init data is taken from daily reports of disease in the open source of web application COVID-19 infestation statistics tracking in real time CSSEGISandData COVID-19 [17]. The volume of representative sets 50 implementations.

Let us consider DSS for COVID-19 caused death prediction in China in 2020 [18]. This system linearizes COVID-19 epidemic spreading. Fifty implementations of random process were investigated. As per given init values in form of statistics in Ukraine on January 1st, 2021, for each learning set implementations formed by this method a root mean square error was evaluated $\varepsilon \in [0.15, 0.35] \approx 28\%$.

Another example of DSS for COVID-19 caused death prediction was suggested in 2021 in [19]. The simulation block of this system uses separable formation method that separates spatial and temporal parts. Let us do the same manipulations with init data taken for Ukraine on January 1st, 2021 and form the learning set by model provided in this DSS. Evaluation or error (8) showing a deviation of simulated process from real one is $\varepsilon \in [0.15, 0.28] \approx 23\%$.

To evaluate the nonlinear discrete dynamical system suggested in this paper let us take init data for Ukraine on January 1st, 2021 and compare real process with the one simulated by nonlinear dynamical system SIR.

The statistics report of January 1st, 2021 was taken as init state vector and passed to system (4-7) input. Based on formed learning set let us compare this model with real process and calculate error (8). We have received normalized root mean square error $\varepsilon \in [0.05, 0.2] \approx 10\%$.

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Thus, a spatial-time non-separable process formation method which uses linearization showed the worst result in prospective of simulation quality. A better result was shown by separable model. However, the best result was shown by the model suggested in this paper which uses nonlinear dynamical systems approach.

CONCLUSIONS

The quasi-periodic spatial-time non-separable process mathematical model has been developed. It was shown in practice that the necessity of a represented learning set formation exists. Developed mathematical model was tested for description of quasi-periodic spatial-time non-separable process of epidemic spreading. The model could be used for learning set formation based on developed mathematical model in DSS construction.

The cellular automata theory was chosen for formation method development because this approach allows describing adequately spatial part of spatial-time process. For temporal part description the methods of dynamical systems theory were chosen allowing providing system state change in time and without requiring the separability of spatial and temporal parts. The formation method of quasi-periodic spatial-time non-separable processes was developed based on mutual usage cellular automata and dynamical systems theory methods.

Comparative and experimental evaluation of such random process as epidemic spreading simulation quality was tested. For this purpose, the volume of representative learning set was defined as N=50. Quality evaluation was provided by normalized root mean square error. The results showed that developed mathematical model and formation method allowed increasing simulation quality by 18 % in comparison with linear model and by 13 % in comparison with separable model.

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Аналіз квазіперіодичних просторово-часових несепарабельних процесів для підтримки прийняття рішень в системах медичного моніторингу

Олена Дмитрівна Франжева
ORCID: https://orcid.org/0000-0003-2639-4012; franzheva.helen@op.edu.ua
Одеський національний політехнічний університет, пр. Шевченка, 1. Одеса, 65044,Україна

АНОТАЦІЯ

В багатьох системах підтримки прийняття рішень обробляються хаотичні просторово-часові процеси, що є несепарабельними та квазіперіодичними. Прикладом таких систем є розповсюдження епідемії, розвиток популяції, розповсюдження пожеж, радіохвильові сигнали, розпізнавання образів, обробка зображень, кодування інформації, радіобачення та ін. Процеси в цих системах мають періодичний характер, наприклад, сезонні коливання (розповсюдження епідемій, зріст популяції), гармонічні коливання (розпізнавання образів, обробка зображень) і т.д. Існуючі системи в блоці моделювання використовують сепарабельні моделі процесів, які представляються у вигляді добутку просторової та часової частини і лінеаризуються, що значно знижує якість моделювання для випадку просторово-часових несепарабельних процесів. Побудова якісної моделі хаотичного просторово-часового несепарабельного процесу, що оброблюється СППР, є необхідністю для отримання навчальної вибірки, так як навчальну вибірку важко зібрати, якщо формується випадковий процес. Ансамбль реалізацій цих процесів потребує великих затрат, що знижує оперативність системи. Більш того, у багатьох випадках ансамбль реалізацій просторово-часових процесів неможливо отримати. В даній роботі розроблено математичну модель квазіперіодичного просторово-часового несепарабельного процесу, на основі якої розроблено та досліджено метод формування цього процесу на прикладі процесу розповсюдження епідемії.

Ключові слова: комп'ютерні системи; прийняття рішень; квазіперіодичні процеси; просторово-часові моделі; несепарабельні процеси; клітинні автомати; динамічні системи; розповсюдження епідемії

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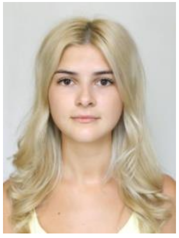

Olena D. Franzheva – PhD student of the Department of Applied Math and Information Technology. Odessa National Polytechnic University, 1, Shevchenko Ave. Odessa, 65044, Ukraine
ORCID: https://orcid.org/0000-0003-2639-4012; franzheva.helen@op.edu.ua
***Research field*:** Information technology; computer science

Олена Дмитрівна Франжева – аспірант каф. Прикладної математики та інформаційних технологій. Одеський національний політехнічний університет, пр. Шевченка, 1. Одеса, 65044,Україна