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Inhomogeneous transversely isotropic space under influence of concentrated power and temperature sources

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Abstract. The problem of constructing fundamental solutions to the thermoelasticity problem for a piecewise-homogeneous transversely isotropic space is reduced to the matrix Riemann problem in the space of generalized slow growth functions. As a result of the solution of which, were obtained expressions in explicit form for the components of the stress tensor and the displacement vector in plane of connection of transversely isotropic elastic half-spaces containing concentrated stationary heat sources. The temperature distribution is investigated depending on the thermophysical characteristics of the half-space materials.

Introduction

The study of stress concentration in the vicinity of interfacial and internal defects such as cracks or inclusions in thermoelastic fields is of great practical importance. Many works have been devoted to this problem for various environments. In particular, in [1-4], the problems of elasticity and thermoelasticity about interfacial stress concentrators such as cracks or rigid inclusions in piecewise homogeneous isotropic and transversely isotropic spaces are considered, which are reduced to systems of two-dimensional singular integral equations (SIE) and proposed a method for their solution.

In the mathematical formulation and solution of such problems about defects, it is necessary to set the boundary conditions on the defect itself, such as stress on the crack edges or displacement at the inclusion. Since in thermophysical formulation of the problems from determining the stress and displacement fields in the vicinity of the stress concentrators, known the stresses or displacements at the boundary of the region, at some interior points or at infinity (for unbounded bodies), then the determination of the boundary conditions on the defect is a separate problem.

Within the framework of the linear theory of thermoelasticity, to solve this problem, it is necessary to know the distribution of the temperature, stress and displacement fields in the corresponding piecewise homogeneous bodies without defects in the presence of volumetric forces and concentrated heat sources.

In particular, for piecewise homogeneous isotropic and transversally isotropic spaces, such solutions are given, respectively, in [5, 6]. Green's functions for piecewise homogeneous transversally isotropic spaces in the presence of a concentrated heat source and in the absence

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of thermal diffusion were constructed in [7–10], and in the presence of thermal diffusion — in [11]. In [12], the Green's function was constructed for a continuous transversely isotropic space with allowance for the orientation of the symmetry axes. For piezo and magnetoelectric compound transversely isotropic spaces, the Green's function was constructed in [13–15]. The fundamental solution for porous transversely isotropic materials was constructed in [16, 17], and for functionally graded materials in [18, 19]. In [20–22], Green's functions for a layered thermal environment were constructed.

In [23–25], similar problems for composite materials were solved by numerical methods.

In this work, using the approach of works [26–28], the problem of constructing fundamental solutions for piecewise homogeneous two-dimensional anisotropic media is reduced to the matrix Riemann problem and obtained its exact solution, which made it possible to construct in an explicit form a fundamental solution for a piecewise-homogeneous transversely isotropic space in the presence of concentrated forces and heat sources.

1. Statement of the problem

Let in an inhomogeneous space composed of two different transversally isotopic half-spaces completely linked in the plane $z = 0$, at an arbitrary point $M_0(x_0, y_0, z_0)$ concentrated force $P = (P_1, P_2, P_3)$ and at an arbitrary point $M_1(x_1, y_1, z_1)$ stationary heat sources.

The thermoelastic state of space is described by the vector

$$\mathbf{v} = \{v_k(x, y, z)\}_{k=\overline{1,9}} = \{\sigma_x, \sigma_y, \sigma_z, \tau_{yz}, \tau_{xz}, \tau_{xy}, u, v, w\}. \tag{1}$$

Based on the equilibrium equations and the generalized Hooke's law, and also taking into account the Duhamel–Neumann relation with respect to the components of the vector $\mathbf{v}$, in the space of generalized functions of slow growth $\Im'(\mathbb{R}^3)$ we write the following boundary value problem

$$\mathbf{D}[z, \partial_1, \partial_2, \partial_3]\mathbf{v} = \mathbf{F}, \quad \mathbf{v}, \mathbf{F} \in \Im'(\mathbb{R}^3), \tag{2}$$

$$v_k(x, y, +0) = v_k(x, y, -0), \quad k = \overline{1,9}, \quad k \neq 1, 2, 6, \tag{3}$$

$$v_k(x, y, z)|_{(x,y,z)\to\infty} = 0, \quad k = \overline{1,9}. \tag{4}$$

Here we use the notation

$$\mathbf{D} = \left\| \begin{matrix} \mathbf{D}_0 & \mathbf{O}_{3\times3} \\ -\mathbf{S} & \mathbf{D}_0^T \end{matrix} \right\|, \quad \mathbf{F}^T = \mathbf{F}_0^T + \mathbf{F}_*^T, \quad \mathbf{F}_0^T = -\delta(x - x_0, x - x_0, x - x_0)\|P_1, P_2, P_3, \mathbf{O}_{1\times6}\|,$$

$$\mathbf{F}_*^T = \|\mathbf{O}_{1\times3}, \beta_1 T, \beta_2 T, \beta_3 T, \mathbf{O}_{1\times3}\|, \quad \mathbf{S} = \left\| \begin{matrix} \mathbf{S}_1 & \mathbf{O}_{3\times3} \\ \mathbf{O}_{3\times3} & \mathbf{S}_2 \end{matrix} \right\|,$$

$$\mathbf{D}_0 = \left\| \begin{matrix} \partial_1 & 0 & 0 & 0 & \partial_3 & \partial_2 \\ 0 & \partial_2 & 0 & \partial_3 & 0 & \partial_1 \\ 0 & 0 & \partial_3 & \partial_2 & \partial_1 & 0 \end{matrix} \right\|, \quad \mathbf{S}_1 = \left\| \begin{matrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{11} & s_{13} \\ s_{13} & s_{13} & s_{33} \end{matrix} \right\|, \quad \mathbf{S}_2 = \left\| \begin{matrix} s_{44} & 0 & 0 \\ 0 & s_{44} & 0 \\ 0 & 0 & s_{66} \end{matrix} \right\|,$$

$\partial_1 = \frac{\partial}{\partial x}$, $\partial_2 = \frac{\partial}{\partial y}$, $\partial_3 = \frac{\partial}{\partial z}$, $s_{kj} = \theta(z)s_{kj}^+ + \theta(-z)s_{kj}^-$, $s_{kj}^\pm$ — of the generalized Hooke's law, respectively, for the upper $z > 0$ and lower $z < 0$ half-spaces; $O_{n\times m}$ is zero matrix of dimension $n \times m$, $\beta_k = \theta(z)\beta_k^+ + \theta(-z)\beta_k^-$, $\beta_k^\pm$ is thermal expansion coefficients, T is temperature from a concentrated heat source with a capacity Q which was received in [13]

$$T(x, y, z) = \frac{m_{31}}{\sqrt{r_1^2 + (\xi_0|z - z_1|)^2}} + \frac{m_{32}}{\sqrt{r_1^2 + (\xi_0(z + z_1))^2}} + \frac{m_{33}}{\sqrt{r_1^2 + (\hat{\xi}_0 z + \check{\xi}_0 z_1)^2}},$$

where

$$m_{31}=\theta(z,z_0)m_{31}^{+}+\theta(-z,-z_0)m_{31}^{-},\quad m_{32}=\theta(-z,-z_0)m_{32}^{-}-\theta(z,z_0)m_{32}^{+},$$
$$m_{33}=\theta(z,-z_0)m_{33}^{+}-\theta(-z,z_0)m_{33}^{-},$$
$$m_{11}^{\pm}=\frac{\lambda_1^{\pm}}{\xi_0^{\pm}},m_{13}^{\pm}=\lambda_1^{\pm}(\lambda_3^{\pm}m_1^{\mp}\pm\frac{m_2^{\mp}}{\xi_0^{\pm}}),\quad m_{12}^{\pm}=\lambda_1^{\pm}(\lambda_3^{\pm}m_1^{\pm}\pm\frac{m_2^{\pm}}{\xi_0^{\pm}}),\quad m_{21}^{\pm}=1,$$
$$m_{22}^{\pm}=\frac{\lambda_1^{\pm}\lambda_3^{+}}{\xi_0^{\pm}}m_1^{\pm}\pm\lambda_3^{\pm}m_2^{\pm},\quad m_{23}^{\pm}=\frac{\lambda_1^{\pm}\lambda_3^{+}}{\xi_0^{\pm}}m_1^{\mp}\pm\lambda_3^{\pm}m_2^{\mp},\quad m_{31}^{\pm}=\frac{1}{\lambda_3^{\pm}\xi_0^{\pm}},$$
$$m_{32}^{\pm}=\lambda_3^{\pm}m_1^{\pm}\pm\frac{m_2^{\pm}}{\xi_0^{\pm}},\quad m_{33}^{\pm}=\lambda_3^{\pm}m_1^{\mp}+\frac{m_2^{\mp}}{\xi_0^{\pm}},\quad \xi_0^{\pm}=\sqrt{\lambda_1^{\pm}/\lambda_3^{\pm}},$$
$$\hat{\xi}_0=\theta(z,z_0)\xi_0^{+}+\theta(z,-z_0)\xi_0^{+}+\theta(-z,-z_0)\xi_0^{-}+\theta(-z,z_0)\xi_0^{-},\quad \xi_0=\theta(z,z_0)\xi_0^{+}+\theta(-z,-z_0)\xi_0^{-},$$
$$\breve{\xi}_0=\theta(z,z_0)\xi_0^{-}-\theta(z,-z_0)\xi_0^{-}+\theta(-z,-z_0)\xi_0^{+}-\theta(-z,z_0)\xi_0^{+},\quad Q=\theta(z,z_1)Q^{+}+\theta(-z,-z_1)Q^{-},$$

$\lambda_i=\lambda_i^{+}\theta(x_3)+\lambda_i^{-}\theta(-x_3)$, $i=\overline{1,3}$, $\lambda_i^{\pm}$ is thermal conductivity coefficient for the upper $z>0$ and lower $z<0$ half-spaces, $r_1=\sqrt{(x-x_1)^2+(y-y_1)^2}$.

2. Construction of the fundamental solution problem of thermoelasticity

We apply to the matrix equation (2) the operator of the three-dimensional Fourier transform F_3 from $\Im'(\mathbb{R}^3)$, given the following representation for vector components $\mathbf{v}$: $v_k=\theta(z)v_k+\theta(-z)v_k=v_k^{+}+v_k^{-}$, where $v_k^{\pm}\in\Im'(\mathbb{R}^3_{\pm})$, $\mathbb{R}^3_{\pm}=\mathbb{R}^2\times\mathbb{R}_{\pm}$. Then, considering the conditions (3), (4) and the results of [8–14], with respect to $V_k^{\pm}(\alpha_1,\alpha_2,\alpha_3)=F_3[v_k^{\pm}]\in\Im'(\mathbb{R}^3)$, and also that the functions $V_k^{\pm}\in\Im'(\mathbb{R}^3_{\pm})$ admit an analytical representation [8, 10] in the variable α_3 we obtain the following matrix equation:

$$\mathbf{M}_{\pm}\mathbf{V}^{\pm}=\mathbf{F}^{\pm},\quad \mathbf{W}^{\pm},\mathbf{F}^{\pm}\in\Im'(\mathbb{R}^3). \tag{5}$$

Here we use the notation

$$\mathbf{M}_{\pm}=\mathbf{D}[\pm0,-i\alpha_1,-i\alpha_2,-i\alpha_3],\quad \mathbf{F}_j^{\pm}=\{f_k^{\pm}\}_{k=\overline{1,4}},\quad f_k^{\pm}=e_0^{\pm}P_k\mp\frac{1}{2}\chi_k,\quad k=1,2,3,$$
$$f_k^{\pm}=\beta_{k-2}^{\pm}T^{\pm}(\alpha_3)\mp\frac{1}{2}\chi_k,\quad k=4,5,6,\quad f_k^{\pm}=\mp\frac{1}{2}\chi_k,\quad k=7,8,\quad f_9^{\pm}=\mp\frac{1}{2}\chi_k,$$
$$\boldsymbol{\chi}=\{\chi_k\}_{k=\overline{1,4}}\in\Im'(\mathbb{R}^2),\quad \chi_k=0,\quad k=4,5,9,\quad e_0^{\pm}=\theta(\pm z_0)\exp(i\alpha_1x_0+i\alpha_2y_0+i\alpha_3z_0),$$
$$T^{\pm}(\alpha_3)=e_0\left\{Q^{\pm}\theta(\pm z_1)\left[\frac{e^{i\alpha_3z_1}}{\alpha_3^2\lambda_3^{+}+\lambda_1^{+}r^2}+\frac{1}{r}(\frac{(-i\alpha_3)\lambda_3^{\pm}m_1^{\pm}-rm_2^{\pm}}{\alpha_3^2\lambda_3^{\pm}+\lambda_1^{\pm}r^2})e^{\mp\xi_0^{+}rz_1}\right]\right.$$
$$\left.-\theta\mp z_1)Q^{\mp}(\frac{(-i\alpha_3)\lambda_3^{\pm}m_1^{\mp}-rm_2^{\mp}}{\alpha_3^2\lambda_3^{\pm}+\lambda_1^{\pm}r^2})e^{\pm\xi_0^{-}rz_1}\right\},\quad e_0=\exp(i\alpha_1x_1+i\alpha_2y_1),$$

$\chi_k(\alpha_1,\alpha_2)$ are unknown functions from $\Im'(\mathbb{R}^2)$ for determine which, we need to use conditions (3) after the Fourier-transformed.

We represent the sought functions as

$$V_7^{\pm}=-(-i\alpha_2)\Psi_1^{\pm}-(-i\alpha_1)\Psi_2^{\pm},\quad V_8^{\pm}=(-i\alpha_1)\Psi_1^{\pm}-(-i\alpha_2)\Psi_2^{\pm}, \tag{6}$$
$$V_5^{\pm}=-(-i\alpha_2)\Upsilon_1^{\pm}-(-i\alpha_1)\Upsilon_2^{\pm},\quad V_4^{\pm}=(-i\alpha_1)\Upsilon_1^{\pm}-(-i\alpha_2)\Upsilon_2^{\pm}, \tag{7}$$

where $\Psi_k^{\pm}$, $\Upsilon_2^{\pm}$, $k=1,2$ are new unknown functions. Then matrix equation (5) can be separated into two independent equations

$$\mathbf{L}_{\pm}\mathbf{U}_1^{\pm}=\mathbf{F}_1^{\pm},\quad \mathbf{G}_{\pm}\mathbf{U}_2^{\pm}=\mathbf{F}_2^{\pm}, \tag{8}$$

where

$$\begin{aligned}
&\mathbf{U}_1^{\pm} = \{U_k^{1,\pm}\}_{k=1,2} = \{\Upsilon_{1j}^{\pm}, \Psi_{1j}^{\pm}\}, \quad \mathbf{U}_2^{\pm} = \{U_k^{2,\pm}\}_{k=1,4} = \{V_3^{\pm}, \Upsilon_2^{\pm}, \Psi_2^{\pm}, V_9^{\pm}\},\\
&\mathbf{F}_1^{\pm} = \{(-i\alpha_2)f_1^{\pm} - (-i\alpha_1)f_2^{\pm}, (-i\alpha_2)f_7^{\pm} - (-i\alpha_1)f_8^{\pm}\},\\
&\mathbf{F}_2^{\pm} = \{f_3^{\pm}, (-i\alpha_1)f_1^{\pm} + (-i\alpha_2)f_2^{\pm}, (-i\alpha_2)f_8^{\pm} + (-i\alpha_1)f_7^{\pm}, f_6^{\pm}\},\\
&\mathbf{G}_{\pm} = \{g_{kj}^{\pm}\}_{k,j=1,\dots,4}, \quad g_{11}^{\pm} = g_{44}^{\pm} = (-i\alpha_3), \quad g_{12}^{\pm} = r^2, \quad g_{21}^{\pm} = -\frac{c_{13}^{\pm}}{c_{33}^{\pm}} g_{12}^{\pm},\\
&g_{23}^{\pm} = -\frac{\bar{c}_{13}^{\pm} + c_{13}^{\pm 2}}{c_{33}^{\pm}} r^4, \quad g_{22}^{\pm} = g_{33}^{\pm} = (-i\alpha_3)r^2, \quad g_{kj}^{\pm} = g_{jk}^{\pm} = 0, \quad k=1,2, \quad j=3,4,\\
&g_{32}^{\pm} = -\frac{1}{c_{44}^{\pm}} g_{12}^{\pm}, \quad g_{34}^{\pm} = -g_{12}^{\pm}, \quad g_{41}^{\pm} = -\frac{1}{c_{33}^{\pm}}, \quad g_{43}^{\pm} = \frac{c_{13}^{\pm}}{c_{33}^{\pm}} g_{12}^{\pm}, \quad r^2 = \alpha_1^2 + \alpha_2^2,\\
&\mathbf{L}_{\pm} = \{l_{kj}^{\pm}\}_{k,j=1,2}, \quad l_{11}^{\pm} = (-i\alpha_3)r^{-2}, \quad l_{22}^{\pm} = (-i\alpha_3)r^2, \quad l_{21}^{\pm} = -\frac{r^2}{c_{44}^{\pm}}, \quad l_{21}^{\pm} = -c_{66}r^4,
\end{aligned}$$

Directly from equations (8) we obtain $\mathbf{U}_1^{\pm} = \mathbf{L}_{\pm}^{-1}\mathbf{F}_1^{\pm}$, $\mathbf{U}_2^{\pm} = \mathbf{G}_{\pm}^{-1}\mathbf{F}_2^{\pm}$, where $\mathbf{L}_{\pm}^{-1} = \{l_{kj}^{*,\pm}\}_{k,j=1,2}$, $\mathbf{G}_{\pm}^{-1} = \{g_{kj}^{*,\pm}\}_{i,j=1,\dots,4}$. Further, using representations (6), (7) after applying the inverse Fourier transform, we obtain such representations for σ_z, τ_{xz}, τ_{yz}:

$$\begin{aligned}
\sigma_z = &-\sum_{n=1}^{3}\Big(\frac{R_{1,n}^0}{\sqrt{r_1^2 + (\xi_n|z-z_1|)^2}} - \frac{\omega_{1,n}}{\sqrt{r_1^2 + (\hat{\xi}_n z + \breve{\xi}_0 z_1)^2}}\Big) + \sum_{n=1}^{2}\sum_{m=1}^{3}\frac{\alpha_{1,n,m}}{\sqrt{r_1^2 + (\hat{\xi}_n z + \breve{\xi}_m z_1)^2}}\\
&+ \sum_{j=1}^{2} P_j \vartheta_{2j}\left\{\sum_{n=1}^{2}\frac{R_{1,2,n}^*}{[r_0^2 + (\xi_n|z-z_0|)^2]^{3/2}} + \sum_{n,m=1}^{2}\frac{\beta_{1,n,m}^2}{[r_0^2 + (\hat{\xi}_n z + \breve{\xi}_m z_0)^2]^{3/2}}\right\}\\
&- P_3\left\{\sum_{n=1}^{2}\frac{|z-z_0|\hat{R}_{1,1,n}^*}{[r_0^2 + (\xi_n|z-z_0|)^2]^{3/2}} - \sum_{n,m=1}^{2}\frac{z\hat{\beta}_{1,n,m}^1 + z_0\breve{\beta}_{1,n,m}^1}{[r_0^2 + (\hat{\xi}_n z + \breve{\xi}_m z_0)^2]^{3/2}}\right\},
\end{aligned} \tag{9}$$

$$\begin{aligned}
\tau_{xz} = &\,\vartheta_{11}\left\{\sum_{n=1}^{3}\frac{R_{2,n}^0[r_0^2 + (\xi_n|z-z_0|)^2]^{-1/2}}{\xi_n|z-z_0| + \sqrt{r_0^2 + (\xi_n|z-z_0|)^2}} - \sum_{n=1}^{3}\frac{\omega_{2,n}[r_0^2 + (\hat{\xi}_n z + \breve{\xi}_0 z_0)^2]^{-1/2}}{\hat{\xi}_n|z| + \breve{\xi}_0|z_0| + \sqrt{r_0^2 + (\hat{\xi}_n z + \breve{\xi}_0 z_0)^2}}\right.\\
&\left. - \sum_{n=1}^{2}\sum_{m=1}^{3}\frac{\alpha_{2,n,m}[r_0^2 + (\hat{\xi}_n z + \breve{\xi}_m z_0)^2]^{-1/2}}{\hat{\xi}_n|z| + \breve{\xi}_m|z_0| + \sqrt{r_0^2 + (\hat{\xi}_n z + \breve{\xi}_m z_0)^2}}\right\}\\
&+ \sum_{j=1}^{2} P_j\left(\partial_2\vartheta_{1j}\left\{\frac{(-1)^{j-1}S_{11}[r_0^2 + (\xi_0^+|z-z_0|)^2]^{-1/2}}{\xi_0^+|z-z_0| + \sqrt{r_0^2 + (\xi_0^+|z-z_0|)^2}} + \frac{(-1)^{j-1}\tilde{\beta}_1[r_0^2 + (\hat{\xi}_0 z + \breve{\xi}_0 z_0)^2]^{-1/2}}{\hat{\xi}_0 z + \breve{\xi}_0 z_0 + \sqrt{r_0^2 + (\hat{\xi}_0 z + \breve{\xi}_0 z_0)^2}}\right\}\right.\\
&\left. + \partial_1\vartheta_{2j}\sum_{n,m=1}^{2}\left\{\frac{-R_{2,2,n}^*[r_0^2 + (\xi_n|z-z_0|)^2]^{-1/2}}{|z-z_0|\xi_n^+ + \sqrt{r_0^2 + (\xi_n|z-z_0|)^2}} + \frac{\beta_{2,n,m}^2[r_0^2 + (\hat{\xi}_n z + \breve{\xi}_m z_0)^2]^{-1/2}}{\hat{\xi}_n|z| + \breve{\xi}_m|z_0| + \sqrt{r_0^2 + (\hat{\xi}_n z + \breve{\xi}_m z_0)^2}}\right\}\right)\\
&- \vartheta_{11}\left\{\sum_{n=1}^{2}\frac{R_{2,1,n}^*}{[r_0^2 + (\xi_n|z-z_0|)^2]^{3/2}} + \sum_{n,m=1}^{2}\frac{\beta_{q,n,m}^1}{[r_0^2 + (\hat{\xi}_n z + \breve{\xi}_m z_0)^2]^{3/2}}\right\},
\end{aligned} \tag{10}$$

$$\tau_{yz} = \vartheta_{12}\left\{\sum_{n=1}^{3}\frac{R^0_{2,n}[r_0^2+(\xi_n|z-z_0|)^2]^{-1/2}}{\xi_n|z-z_0|+\sqrt{r_0^2+(\xi_n|z-z_0|)^2}} - \sum_{n=1}^{3}\frac{\omega_{2,n}[r_0^2+(\hat{\xi}_n z+\breve{\xi}_0 z_0)^2]^{-1/2}}{\hat{\xi}_n|z|+\breve{\xi}_0|z_0|+\sqrt{r_0^2+(\hat{\xi}_n z+\breve{\xi}_0 z_0)^2}}\right.$$

$$\left.-\sum_{n=1}^{2}\sum_{m=1}^{3}\frac{\alpha_{2,n,m}[r_0^2+(\hat{\xi}_n z+\breve{\xi}_m z_0)^2]^{-1/2}}{\hat{\xi}_n|z|+\breve{\xi}_m|z_0|+\sqrt{r_0^2+(\hat{\xi}_n z+\breve{\xi}_m z_0)^2}}\right\}$$

$$+\sum_{j=1}^{2}P_j\left(\partial_1\vartheta_{1j}\left\{\frac{(-1)^{j-1}S_{11}[r_0^2+(\xi_0^+|z-z_0|)^2]^{-1/2}}{\xi_0^+|z-z_0|+\sqrt{r_0^2+(\xi_0^+|z-z_0|)^2}}+\frac{(-1)^{j-1}\tilde{\beta}_1[r_0^2+(\hat{\xi}_0 z+\breve{\xi}_0 z_0)^2]^{-1/2}}{\hat{\xi}_0 z+\breve{\xi}_0 z_0+\sqrt{r_0^2+(\hat{\xi}_0 z+\breve{\xi}_0 z_0)^2}}\right\}\right.$$

$$\left.+\partial_2\vartheta_{2j}\sum_{n,m=1}^{2}\left\{\frac{-R^*_{2,2,n}[r_0^2+(\xi_n|z-z_0|)^2]^{-1/2}}{|z-z_0|\xi_n^+ +\sqrt{r_0^2+(\xi_n|z-z_0|)^2}}+\frac{\beta^2_{2,n,m}[r_0^2+(\hat{\xi}_n z+\breve{\xi}_m z_0)^2]^{-1/2}}{\hat{\xi}_n|z|+\breve{\xi}_m|z_0|+\sqrt{r_0^2+(\hat{\xi}_n z+\breve{\xi}_m z_0)^2}}\right\}\right)$$

$$+\vartheta_{12}\left\{\sum_{n=1}^{2}\frac{R^*_{2,1,n}}{[r_0^2+(\xi_n|z-z_0|)^2]^{3/2}}+\sum_{n,m=1}^{2}\frac{\beta^1_{q,n,m}}{[r_0^2+(\hat{\xi}_n z+\breve{\xi}_m z_0)^2]^{3/2}}\right\}, \tag{11}$$

where

$$q_j^{\pm}(\alpha_3,r) = g^{*,\pm}_{j2}(\alpha_3,r)\gamma_1^{\pm}r^2+g^{*,\pm}_{j4}(\alpha_3,r)\gamma_2^{\pm}, \quad j=\overline{1,4},$$

$$R^{0,+}_{1,n} = \frac{q_1^+(-i\xi_n^+,1)\tau^{++}(-i\xi_n^+,1)}{\xi_n^+h_n^+}, \quad \bar{R}^{0,+}_{1,n} = \frac{q_1^+(i\xi_n^+,1)\tau^{++}(i\xi_n^+,1)}{\xi_n^+h_n^+},$$

$$\tau^{++}(\alpha_3,r) = (-i\alpha_3)\lambda_3^+m_1^+-rm_2^+, \quad \tau^{-+}(\alpha_3,r) = (-i\alpha_3)\lambda_3^-m_1^+-rm_2^+,$$

$$\tau^{+-}(\alpha_3,r) = (-i\alpha_3)\lambda_3^+m_1^--rm_2^-, \quad \tilde{\beta}^{-+}_{1,n} = \frac{q_1^-(i\xi_n^{\pm},1)\tau^{-+}(i\xi_n^-,1)}{\xi_n^-h_n^-},$$

$$\tilde{\beta}^{+-}_{1,n} = \frac{q_1^+(-i\xi_n^+,1)\tau^{+-}(-i\xi_n^+,1)}{\xi_n^+h_n^+}, \quad \tilde{\beta}^{++}_{j,n} = \frac{q_j^+(-i\xi_n^+,1)\tau^{++}(-i\xi_n,1)}{\xi_n^+h_n^+}, \quad j=\overline{1,4},$$

$$h_n^{\pm} = \prod_{l=1,l\neq n}^{3}(\xi_n^{\pm})^2-(\xi_l^{\pm})^2, \quad \tau^{--}(\alpha_3,r) = (-i\alpha_3)\lambda_3^-m_1^--rm_2^-,$$

$$\tilde{\beta}^{--}_{1,n} = \frac{q_1^-(i\xi_n^-,1)\tau^{--}(i\xi_n^-,1)}{\xi_n^-h_n^-}, \quad R^{\pm}_{j,k} = \sum_{n=1}^{2}R^{*,+}_{j,k,n}, \quad \beta_j^{\pm\pm} = \sum_{n=1}^{3}\tilde{\beta}^{\pm\pm}_{j,n}, \quad \beta_j^{\pm\mp} = \sum_{n=1}^{3}\tilde{\beta}^{\pm\mp}_{j,n},$$

$$\mathbf{A}_0^{-1} = \{a^*_{kj}\}_{k,j=\overline{1,4}}, \quad \mathbf{A}_0 = \{a_{k,j}\}_{k,j=\overline{1,4}} = \mathbf{N}^+ + \bar{\mathbf{N}}^-, \quad \mathbf{N}^{\pm} = \{R^{\pm}_{j,k}\}_{k,j=\overline{1,4}},$$

$$\alpha^+_{j,m} = \sum_{k=1}^{4}a^*_{jk}\bar{R}^{0,+}_{k,n}, \quad \alpha^-_{j,m} = \sum_{k=1}^{4}a^*_{jk}R^{0,-}_{k,m}, \quad \mu_j^+ = \sum_{k=1}^{4}a^*_{jk}\beta_k^+, \quad \mu_j^- = \sum_{k=1}^{4}a^*_{jk}\beta_k^-,$$

$$\beta_j^+ = \beta_j^{++}+\beta_j^{-+}, \quad \beta_j^- = \beta_j^{--}+\beta_j^{+-}, \quad \vartheta_{1j} = \frac{(y-y_0)^{2-j}}{(x-x_0)^{1-j}}, \quad \vartheta_{2j} = \frac{(x-x_0)^{2-j}}{(y-y_0)^{1-j}}$$

$$\alpha^{++}_{j,n,m} = \sum_{k=1}^{4}R^{*,+}_{j,k,n}\alpha^+_{k,m}, \quad \alpha^{+-}_{j,n,m} = \sum_{k=1}^{4}R^{*,+}_{j,k,n}\alpha^+_{k,m},$$

$$\alpha^{--}_{j,n,m} = \sum_{k=1}^{4}R^{*,-}_{j,k,n}\alpha^+_{k,m}, \quad \alpha^{-+}_{j,n,m} = \sum_{k=1}^{4}R^{*,-}_{j,k,n}\alpha^+_{k,m},$$

$$\mu_j^{++} = \sum_{k=1}^{4}R^{*,+}_{j,k,n}\mu_j^+, \qquad \mu^{+-}_{j,n} = \sum_{k=1}^{4}R^{*,+}_{j,k,n}\mu_j^-, \quad n=1,2, \qquad \mu^{+-}_{j,n}=0, \quad n=3,$$

$$\mu_{j,n}^{--}=\sum_{k=1}^{4}R_{j,k,n}^{*,-}\mu_j^{-},\qquad \mu_{j,n}^{-+}=\sum_{k=1}^{4}R_{j,k,n}^{*,-}\mu_j^{+},\quad n=1,2,\qquad \mu_{j,n}^{--}=\mu_{j,n}^{-+}=0,\quad n=3,$$
$$\tilde{\omega}_{j,n}^{\pm\pm}=\tilde{\beta}_{j,n}^{\pm\pm}-\mu_{j,n}^{\pm\pm},\quad \tilde{\omega}_{j,n}^{\pm\mp}=\tilde{\beta}_{j,n}^{\pm\mp}-\mu_{j,n}^{\pm\mp},$$
$$\hat{R}_{j,n}^{0,+}=\theta(z-z_0)R_{j,n}^{0,+}+\theta(z_0-z)\bar{R}_{j,n}^{0,+},\quad \breve{R}_{j,n}^{0,-}=\theta(z-z_0)R_{j,n}^{0,-}+\theta(z_0-z)\bar{R}_{j,n}^{0,-}.$$

3. Stress fields in the plane of connection of half-spaces

Putting $z=0$ in the obtained expressions (9)–(11), we obtain the distribution of normal and tangential stresses in the plane of connection of half-spaces in the presence of a stationary source of heat of the average force:

$$\sigma_z(x,y)=-\sum_{n=1}^{3}\frac{R_{1,n}^{0}}{\sqrt{r_0^2+(\xi_n z_0)^2}}+\frac{\omega_1}{\sqrt{r_0^2+(\xi_0 z_0)^2}}+\sum_{n=1}^{3}\frac{\alpha_{1,n}}{\sqrt{r_0^2+(\xi_n z_0)^2}}$$
$$+\sum_{j=1}^{2}P_j\sum_{n=1}^{2}B_{1,n}\frac{\vartheta_{2j}z_0}{[r_0^2+(\xi_n z_0)^2]^{3/2}}-P_3\sum_{n=1}^{2}\frac{A_{1,n}z_0}{[r_0^2+(\xi_n z_0)^2]^{3/2}},$$

$$\tau_{xz}(x,y)=\vartheta_{11}\left\{\sum_{n=1}^{3}\frac{R_{2,n}^{0}[r_1^2+(\xi_n z_1)^2]^{-1/2}}{\xi_n|z_1|+\sqrt{r_1^2+(\xi_n z_1)^2}}-\frac{\omega_2[r_1^2+(\xi_1 z_0)^2]^{-1/2}}{\breve{\xi}_0|z_0|+\sqrt{r_1^2+(\xi_0 z_1)^2}}\right.$$
$$\left.-\sum_{n=1}^{3}\frac{\alpha_{2,n}[r_1^2+(\xi_n z_1)^2]^{-1/2}}{[\xi_n|z_1|+\sqrt{r_1^2+(\xi_n z_1)^2}]}\right\}-\sum_{j=1}^{2}P_j\left\{\partial_2\frac{\vartheta_{1j}(-1)^jS_1[r_0^2+(\xi z_0)^2]^{-1/2}}{\xi|z_0|+\sqrt{r_0^2+(\xi z_0)^2}}\right.$$
$$\left.+\partial_1\sum_{n=1}^{2}\frac{\vartheta_{2j}B_{2,n}[r_0^2+(\xi_n z_0)^2]^{-1/2}}{\xi_n|z_0|+\sqrt{r_0^2+(\xi_n z_0)^2}}\right\}+P_3\sum_{n=1}^{2}\frac{A_{2,n}(x-x_0)}{[r_0^2+(\xi_n z_0)^2]^{3/2}},$$

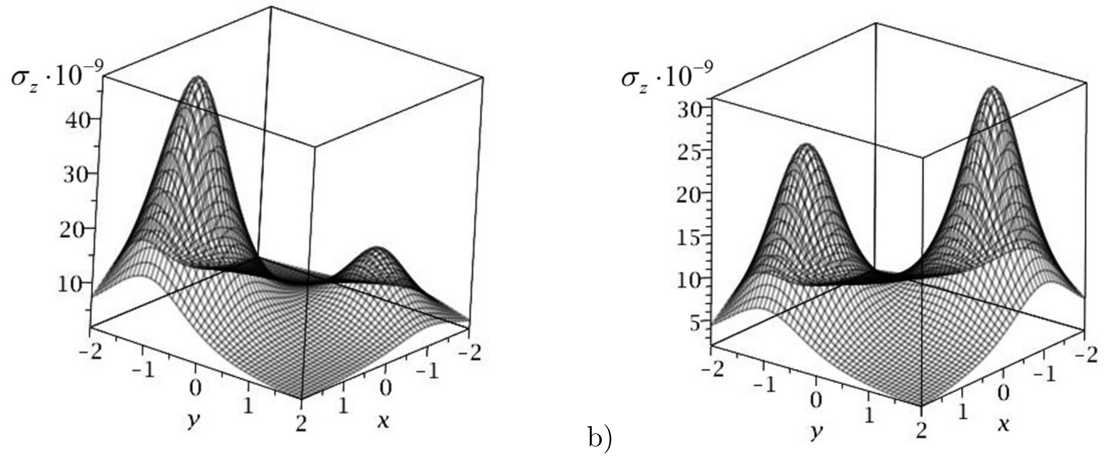
$$\tau_{yz}(x,y)=\vartheta_{12}\left\{\sum_{n=1}^{3}\frac{R_{2,n}^{0}[r_0^2+(\xi_n z_0)^2]^{-1/2}}{\xi_n|z_0|+\sqrt{r_0^2+(\xi_n z_0)^2}}-\frac{\omega_2[r_0^2+(\breve{\xi}_0 z_0)^2]^{-1/2}}{\breve{\xi}_0|z_0|+\sqrt{r_0^2+(\breve{\xi}_0 z_0)^2}}\right.$$
$$\left.-\sum_{n=1}^{3}\frac{\alpha_{2,n}[r_0^2+(\breve{\xi}_n z_0)^2]^{-1/2}}{\breve{\xi}_n|z_0|+\sqrt{r_0^2+(\breve{\xi}_n z_0)^2}}\right\}-\sum_{j=1}^{2}P_j\left\{-\partial_1\frac{\vartheta_{1j}(-1)^{3-j}[r_0^2+(\xi z_0)^2]^{-1/2}S_1}{\xi|z_0|+\sqrt{r_0^2+(\xi z_0)^2}}\right.$$
$$\left.+\partial_2\sum_{n=1}^{2}\frac{\vartheta_{2j}B_{2,n}[r_0^2+(\xi_n z_0)^2]^{-1/2}}{\xi_n|z_0|+\sqrt{r_0^2+(\xi_n z_0)^2}}\right\}+P_3\sum_{n=1}^{2}\frac{A_{2,n}(y-y_0)}{[r_0^2+(\xi_n z_0)^2]^{3/2}},$$

where

$$S_p=\theta(z_0)S_{p1}^{+}+\theta(-z_0)S_{p1}^{-},\quad A_{p,n}=\theta(z_0)A_{p,n}^{+}+\theta(-z_0)A_{p,n}^{-},\quad p=1,2,$$
$$B_{p,n}=\theta(z_0)B_{p,n}^{+}+\theta(-z_0)B_{p,n}^{-},\quad A_{1,n}^{+}=-\hat{R}_n^{+}+\hat{\beta}_n^{++},\quad A_{1,n}^{-}=-\breve{\beta}_n^{+-},$$
$$\hat{\beta}_n^{\pm\pm}=\sum_{m=1}^{2}\hat{\beta}_{m,n}^{\pm\pm},\quad \breve{\beta}_n^{\pm\mp}=\sum_{m=1}^{2}\breve{\beta}_{m,n}^{\pm\mp},\quad \beta_{p,k,n}^{\pm\pm}=\sum_{m=1}^{2}\beta_{k,m,n}^{p,\pm\pm},\quad \beta_{p,k,n}^{\pm\mp}=\sum_{m=1}^{2}\beta_{k,m,n}^{p,\pm\mp},$$
$$A_{k,n}^{+}=-R_{k,1,n}^{-}+\beta_{1,k,n}^{++},\quad A_{k,n}^{-}=-\beta_{1,k,n}^{+-},\quad k=2,3,4,$$
$$B_{k,n}^{+}=-R_{k,2,n}^{+}+\beta_{2,k,n}^{++},\quad B_{k,n}^{-}=-\beta_{2,k,n}^{+-},\quad k=1,\ldots,4.$$

4. Discussion and numerical results

Numerical studies of the stress distribution were carried out depending on the thermophysical properties and the presence at the points $M_1(1,-1,1)$ and $M_2(-1,1,-1)$ stationary concentrated



a) b)

Figure 1. Normal stress distribution for material combination m1–m2.

heat sources with a capacity $Q_1 = 2 \times 10^4$ J and $Q_2 = 10^4$ J. Figures 1 and 2 show the distribution of normal stresses σ_z, in plane $z0y$ for some combinations of transversely isotropic materials. Calculations were performed for the combination of materials: cadmium (material $m1$), magnesium (material $m2$).

Table 1 shows the values of thermoelastic constants of these materials and table 2 shows the attitudes of thermoelastic constants.

Table 1. Material properties of transversely isotropic materials.

	$c_{ij} \times 10^{11}$, N/m^2						$\alpha_j \times 10^{-5}$, (°C)$^{-1}$		λ_j, W/m·°C	
Material	c_{11}	c_{12}	c_{13}	c_{33}	c_{44}	c_{55}	$\alpha_1 = \alpha_2$	α_3	$\lambda_1 = \lambda_2$	λ_3
$m1$	1.08	0.389	0.375	0.46	0.156	0.156	54	20.2	93	94
$m2$	0.5952	0.256	0.214	0.6147	0.1647	0.1647	27.7	20.2	159	160

Table 2. Attitudes of thermoelastic constants.

Material combination	$\zeta_{ij} = c_{ij}^{+}/c_{ij}^{-}$						$\alpha_1^{+}/\alpha_1^{-}$	$\alpha_3^{+}/\alpha_3^{-}$	$\lambda_1^{+}/\lambda_1^{-}$	$\lambda_3^{+}/\lambda_3^{-}$
$m1$–$m2$	1.814	1.519	1.752	0.748	0.947	0.947	1.949	1	0.596	0.587
$m2$–$m1$	0.551	0.658	0.571	1.336	1.055	1.055	0.513	1	1.677	1.702

In figures $1a$ and $2b$ show the distribution of normal stresses when the power of the heat source in the upper half-space is greater than in the lowerand in figures $1a$ and $2b$ vice versa. As can be seen from the above figures, the ratio of the values of the thermoelastic constants of half-spaces has a significant effect on the distribution of stresses near the inclusion, in particular, the value of the coefficient $\zeta_{33} = A_{33}^{+}/A_{33}^{-}$, and attitudes of the thermal conductivity coefficients $\lambda_3^{+}/\lambda_3^{-}$ which characterizes the difference between the elastic and thermal properties of half-spaces in the direction of the axis z. This results in both a qualitative and quantitative change in the distribution of the normal stresses, as evidenced, for example, by comparing the graphs of figures.

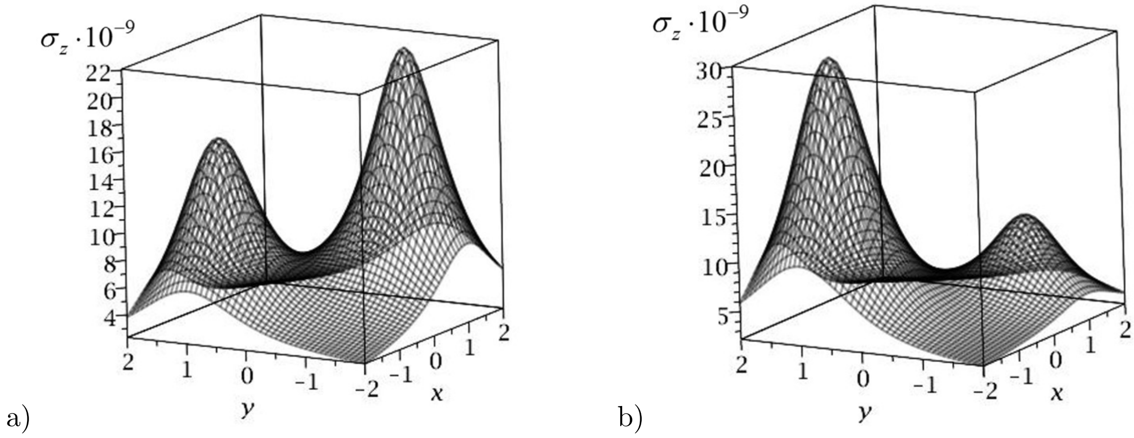


Figure 2. Normal stress distribution for material combination m2–m1.

Conclusion

Built fundamental solution of the thermoelasticity problem for a piecewise-homogeneous transversely isotropic space is constructed in an explicit form, which made it possible to study how temperature affects the stress distribution in the plane of joining materials. In particular, it is shown that the inhomogeneity of the thermoelastic characteristics of the medium significantly affects the stress distribution in space.

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