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AN ALTERNATIVE APPROACH TO CONTROL THE SHAPING OF PARTS WITH SPATIALLY COMPLEX SURFACES

С.А. Зелинський,. А.Ж. Ткач. **Альтернативний підхід управління формоутворенням деталей з просторово-складними поверхнями.** У сучасному машинобудуванні зростає кількість деталей з просторово-складними поверхнями, форму яких визначають аналітично за певними критеріями. Найбільшого поширення вони набули в енергетичному, авіаційному, інструментальному виробництвах (лопаті турбін, моноколеса). При існуючій практиці обробки таких деталей на верстатах з ЧПУ розробки керуючих програм, аналітично- розраховану форму поверхонь замінюють наближеною графічної моделлю, по якої виконують розрахунок траєкторій руху інструмента. Така практика обумовлена історично існуючими у 1970-90 роках можливостями електроніки та електроприводних пристроїв. Незважаючи на принципово нові сучасні можливості комп'ютерного управління та електроприводних пристроїв у верстатах з ЧПУ зберігся традиційний первісний принцип формоутворення поверхонь, з використанням методів апроксимації та інтерполяції. При цьому вже на етапі технологічної підготовки керуючих програм припускають певні припущення, втрати точності. Вирішення цієї проблеми актуальне у зв'язку з існуючою тенденцією зростання вимог до точності відповідальних деталей із просторово складними поверхнями. У статті запропоновано принцип альтернативного управління формотворчими подачами при фрезеруванні деталей із просторово-складними поверхнями при використанні форми завдання поверхні в аналітичному вигляді. Застосування такого принципу виключає проміжні етапи, пов'язані з перетворенням аналітичної форми завдання на графічну, вибором координат опорних точок та інтерполяцією елементарних ділянок. Пропонується, на основі запропонованого функціонального взаємозв'язку між геометричною формою просторово складної поверхні та встановленим співвідношенням складових контурної подачі на базі використання сучасних можливостей обчислювальних пристроїв та електроприводу забезпечити принципово новий підхід формоутворення. Використання запропонованого підходу забезпечує повну автоматизацію підготовки керуючих програм.
Ключові слова: деталі з просторово-складними поверхнями, фрезерування, аналітична модель, похибка, точність, контурна подача

S. Zelinsky, A. Tkach. **An alternative approach to control the shaping of parts with spatially complex surfaces.** In modern mechanical engineering, the number of parts with spatially complex surfaces is growing, the shape of which is determined analytically according to certain criteria. They are most widely used in the energy, aviation, tool industries (turbine blades, unicycles). With the current practice of processing such parts on CNC machines, when developing control programs, the analytically calculated shape of the surfaces is replaced by an approximate graphical model, which is used to calculate tool trajectories. This practice is due to the historically existing in the 1970-90s the capabilities of electronics and electrically driven devices. Despite the fundamentally new modern possibilities of computer control and electric drive devices, CNC machines retained the traditional initial principle of surface shaping using approximation and interpolation methods. At the same time, already at the stage of technological preparation of control programs, certain assumptions and losses in accuracy are assumed. The solution of this problem is relevant in connection with the current trend of increasing requirements for the accuracy of critical parts with spatially complex surfaces. The article proposes the principle of alternative control of shaping feeds when milling parts with spatially complex surfaces using the form of specifying the surface in an analytical form. The application of this principle excludes intermediate stages associated with the transformation of the analytical form of the task into a graphic one, the choice of coordinates of reference points and the interpolation of elementary sections. It is proposed, based on the proposed functional relationship between the geometric shape of a spatially complex surface and the established ratio of the components of the contour feed, based on the use of modern capabilities of computing devices and an electric drive, to provide a fundamentally new approach to shaping. The use of the proposed approach ensures complete automation of the preparation of control programs.
Keywords: parts with spatially complex surfaces (SCS), milling, analytical model, error, accuracy, contour feed

1. Introduction

Processing of parts with SCS is most often carried out on milling machines with numerical control. With the current practice of developing a control program for CNC machines, information about the geometric shape of the SCS, given in the form of a mathematical model, has to be converted into a graphical form in the form of a drawing. In this case, the SCS parts is represented by a set of coordinates of reference points. Straight lines or arcs of circles approximate the nominal surface of the part. In CNC machines, shaping is provided as a result of alternately moving the tool relative to the workpiece by a certain number of increments along the X, Y, Z axes of the machine.

Thus, the underlying principles of shaping in CNC machines suggest the appearance of errors at the stage of developing control programs, caused by rounding the coordinates of reference points, approximation and interpolation. The total value of the listed errors is significant and can reach 10...20 percent of the machining tolerance. In other words, the listed errors reduce the accuracy of machined parts with a SCS by about 1 quality. Therefore, in order to achieve the inherent accuracy of the part, the requirements for their accuracy must be overestimated. In the case of manufacturing high-precision parts, this may not be achievable.

Therefore, the search for new ways to improve the accuracy and efficiency of processing parts with spatially complex surfaces is relevant.

2. Analysis of literature data and statement of the problem

Trends in the development of modern mechanical engineering suggest an improvement in metal consumption and a reduction in the dimensions of structures. This is achieved by using a more complex part that replaces a multi-part assembly, resulting in a reduction in the total number of parts in the machine. This allows you to significantly improve the operational and technical and economic performance of the machine as a whole [1, 2]. This trend is most noticeable in aviation and power engineering. For example, the use of monowheels instead of traditional turbines, consisting of individual blades fixed in the turbine housing. Such parts often belong to the class of parts with spatially complex surfaces (SCS).

Often such parts determine the performance of the machine as a whole and therefore they are subject to increased requirements for accuracy. The shape of the surfaces of such parts is often calculated analytically from the conditions for ensuring optimal performance of the unit or the entire machine (for example, the shape of turbine blades, monowheels, cams, etc.). Therefore, often the initial model of such parts is presented not in a traditional graphical form, but in an analytical one, and is calculated using computer simulation methods [3, 4].

When the geometric shapes and dimensions of the profile deviate from the calculated ones, for example, blades, the gas-dynamic stability of the engine deteriorates, aerodynamic losses increase, leading to a decrease in the efficiency of the turbine, to loss of power, an increase in specific costs, and as a result, to a decrease in the economic performance of the engine. Therefore, the manufacture of the profile part of the blades in strict accordance with the model calculated in the analytical form is a necessary condition for the performance, efficiency and reliability of gas turbine engines.

With the current practice of developing control programs for processing on machine tools with numerical control (CNC), the analytical view of parts with SCS is converted into a graphical form. Naturally, this stage is quite laborious and is associated with a loss of accuracy, which was originally incorporated in the analytical model of the part with the SCS. At this stage, the decrease in accuracy is due to the replacement of the analytical model of the part with a geometric model due to the approximate approximation of its shape between the reference points.

The most time-consuming task in the processing of parts with SCS, taking into account the specified requirements for accuracy and productivity, is the solution of trajectory problems to ensure the shaping of the SCS [5, 6].

With 3-axis contouring on CNC milling machines, shaping is achieved through sequential processing of a spatial part in the $X0Y$, $X0Z$ and $Y0Z$ planes of the machine. The development of a control program for a CNC machine involves the implementation of certain steps for setting reference points and calculating their coordinates.

The trajectory of movement between the reference points of the part is set by the interpolator in the form of discrete movements along the X, Y, Z axes of the machine. Depending on the type of interpolation, movements can only occur along a straight line, a circular arc, or a parabola.

All of the above steps associated with the transformation of the analytical model of the SCS part into a traditional graphic one lead to an increase in labor intensity and the introduction of additional errors, which subsequently leads to a decrease in processing accuracy.

3. Goals and objectives of the study

This article explores the theoretical foundations of a new alternative option for controlling the process of shaping parts with a SCS, in which it is possible to use information about the geometric

shape of a part with a SCS given in the original analytical form. The advantages of this approach in controlling the process of shaping parts with SCS are:

– the intermediate stage of converting the analytical model of the SCS part into a traditional graphical form is excluded and, as a result, the reduction in the complexity of the programming process and the elimination of associated errors in calculating the coordinates of reference points, applying approximation and other procedures;

– additional opportunities are opened for the implementation of the process of spatial shaping of the SCS of higher degrees of curvature than circular and parabolic interpolation.

To achieve this goal, it is proposed to control the shaping process based on the establishment of a functional relationship between the feeds along the machine axes and the geometric shape of parts with a PSP given by an analytical model at each processing point.

Therefore, to solve trajectory problems, a functional relationship must be established between the shaping feeds along the coordinate axes of the machine with the geometric shape of the part with the PSP specified in the analytical form

4. Materials and methods of research

We will divide the solution of the above problem into two stages:

– we will establish a functional relationship between the component feeds in the direction of the coordinate axes of the machine when processing flat contours of a complex curvilinear shape;

– we will establish a functional relationship between the component feeds in the direction of the coordinate axes of the machine when processing spatially complex surfaces.

A theoretical analysis in order to establish a functional relationship between the components of the feeds in the direction of the coordinate axes of the machine tool when processing parts with SCS will be carried out using the methods of analytical geometry and differential calculus.

4.1 Establishing a functional relationship between the components of the contour feed on the machine when processing flat contours of complex curvilinear shape

At the first stage, we will establish a functional relationship of feeds along the coordinate axes of the machine with the geometric shape of the part using the example of processing flat contours of arbitrary shape, given by an equation of the form $Y = f(X)$ [7].

The motion path (by analogy with the kinematic method of processing using a template) can be obtained as a result of two interconnected feeds:

– the first is the master feed S_X, suppose that it is directed along the X axis of the machine;

– the second one is the follow-up feed S_Y, directed along the Y-axis of the machine, Fig. 1.

When the S_X feed is performed, the X coordinate changes continuously, i.e. the X coordinate is a function of time: $X = \varphi(t)$.

Fig. 1. Curve shaping by kinematic method

To obtain a $Y = f(X)$ curve, the Y coordinate must depend on time as follows:

$$Y = f(\varphi(t)). \tag{1}$$

Differentiate the resulting expression with respect to time t:

$$Y'_t = f'[\varphi(t)\varphi'(t)], \tag{2}$$

and rewrite expression (2) taking into account the fact that $Y'(t) = S_Y$; $\varphi(t) = X$; $\varphi'(t)$; $\varphi'(t) = X'_t = S_X$

$$S_Y = f'(X)S_X, \tag{3}$$

where $f'(X)$ – the value of the first derivative of the function describing contour in the machine's rectangular coordinate system.
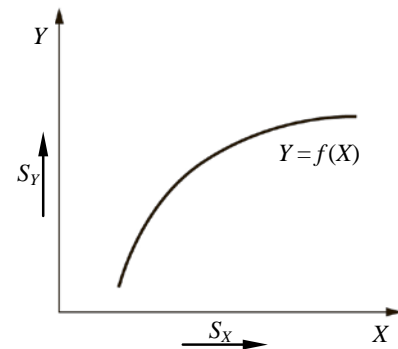
In accordance with expression (3), for the formation of a contour, the ratio of feed rates at each processing point must satisfy the condition: $S_Y/S_X = f'(X)$.

In this case, the feeds S_X and S_Y along the machine axes X and Y must be carried out simultaneously.

4.2 Establishment of a functional relationship between the components of the contour feed in the processing of spatially complex surfaces

Let us establish a functional relationship between feeds when processing an arbitrary SCS given by an equation of the form [8]:

$$Z = \psi(X,Y).\tag{4}$$

The shaping of a volumetric surface differs from the flat case described above by the presence of a feed directed along the third axis of the machine – the Z axis.

Let us differentiate (4) with respect to time t:

$$\frac{dZ}{dt} = \frac{\partial Z}{\partial X} \cdot \frac{dX}{dt} + \frac{\partial Z}{\partial Y} \cdot \frac{dY}{dt},\tag{5}$$

and rewrite the resulting expression, taking into account the fact that $\frac{dZ}{dt} = S_z$, $\frac{dX}{dt} = S_x$, $\frac{dY}{dt} = S_y$:

$$S_Z = \frac{\partial Z}{\partial X} S_X + \frac{\partial Z}{\partial Y} S_Y,\tag{6}$$

where, S_Z – amount of feed in the direction of the Z-axis of the machine;

$\frac{\partial Z}{\partial X}$ and $\frac{\partial Z}{\partial Y}$ – partial derivatives of the Z coordinate with respect to the X and Y coordinates.

Expression (6), by determining the value of S_Z at any point of processing, includes the values of independent feeds S_X and S_Y, which in this case will be master, and the feed S_Z will be follow-up. If feeds S_X and S_Y are interconnected by the equation $S_Y = f(X)S_X$, then the feed rate S_Z is determined as follows:

$$S_Z = \left(\frac{\partial Z}{\partial X} + \frac{\partial Z}{\partial Y} \cdot \frac{dY}{dX}\right)S_X.\tag{7}$$

For shaping a surface of arbitrary geometric shape, given by an equation of the form $Z = \psi(X,Y)$, the functional relationship of feeds along the coordinate axes of the machine for the part SCS can be written as a system:

$$\begin{cases} S_Y = f'(x)S_x; \\ S_Z = \left(\dfrac{\partial Z}{\partial X} + \dfrac{\partial Z}{\partial Y} \cdot \dfrac{dY}{dX}\right)S_X. \end{cases}\tag{8}$$

In this case, the S_X feed is the master feed, and the S_Y and S_Z feeds are the tracking feeds.

In accordance with expression (8), the cutting tool (cutter) will describe spatial curves relative to the workpiece, the totality of which forms a surface given by a mathematical equation of the form $Z = \psi(X,Y)$.

Thus, the task of establishing a functional relationship between the components of the contour feed in the processing of flat curvilinear contours and spatially complex surfaces has been solved.

5. Investigation of ways to improve accuracy for the practical implementation of the proposed method for shaping parts with SCS

The established relationship between the geometric shape of the SCS and the components of the contour feed makes it possible to obtain a shaping trajectory that ensures that it coincides with the

nominal surface at all points. This can be achieved under the condition of a continuous change in the ratio of feed rates, in accordance with expression (8), at each processing point.

The characteristics of technical devices can provide the possibility of implementing the developed principle of shaping when changing the ratio of the speeds of the feed components at certain intervals at certain points of the trajectory, which are located at a certain distance from each other.

In this case, the processing path will consist of straight sections parallel to the tangents to the nominal surface at the points of change in the ratios of the speeds of the components of the contour feed. As a result, there is an error associated with the deviation of the machining path from the nominal surface. A graphical interpretation of the formation of errors for flat machining is shown in Fig. 2.
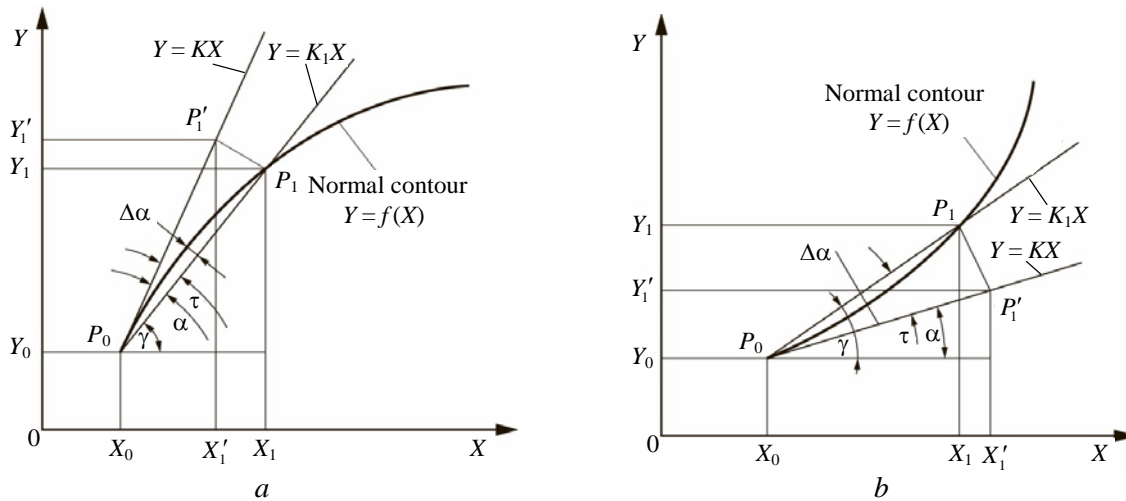

Fig. 2. Scheme of the formation of errors of deviation from the nominal contour
at the reference points: *a* – convex; *b* – concave

To improve the accuracy of shaping with a discrete change in the ratio of feed rates, it is necessary to compensate for the resulting errors. To find ways to compensate for errors, consider the patterns of their formation using the example of processing flat curvilinear contours. Graphically, this is shown in Fig. 2, *a* – for a convex contour and 2, *b* – for a concave one.

5.1 Increasing the accuracy of shaping by correcting the ratio of the speeds of the components of the contour feed when processing flat contours of arbitrary shape

Let the point P_0 with coordinates (X_0, Y_0) be the beginning of contour processing $Y = f(X)$. The ratio of feed rates at point P_0 is determined by:

$$\frac{S_{Y0}}{S_{X0}} = f'(X_0),$$

where $f'(X_0)$ – the value of the first derivative, numerically equal to the tangent of the angle of inclination of the tangent to the axis *X*.

The next change in the ratio will occur at the point P_1, which is at a distance from the point P_0 at the distance of the approximation step – Δl.

Due to the fact that the trajectory of movement is carried out along a tangent, the section of the curve $P_0 P_1$ is replaced by a segment of the tangent $P_0 P_1$ which can be described by the equation:

$$Y = KX, \text{ there } K = f'(X_0) = \text{tg}\,\alpha.$$

The error of the trajectory of processing arises from the fact that the value of $f'(X)$ is assumed to be constant for the entire section of the curve $P_0 P$ enclosed between the reference points. In fact, it changes at each point of the contour [9].

As a result, the curvilinear section of the nominal trajectory $P_0 P$ is replaced by a section of the tangent $P_0 P'_1$, which leads to the appearance of an error $P_1 P'_1$ – a deviation from the nominal contour

given by the equation $Y = f(X)$. This error can be corrected by replacing the machining path along the tangent P_0P_1' with the machining path along the chord P_0P_1. To do this, it is necessary to adjust the ratio of feed rates at point P_0. Graphically, this can be represented as an additional rotation of the tangent P_0P_1' at the point P_0 by an angle τ.

Correction angle τ (adjustment angle) is defined as the difference between the angles of inclination of the chord and the tangent:

$$\tau = \gamma - \alpha . \tag{9}$$

For a convex contour $\tau < 0$, for a concave one $\tau > 0$.

The correction angle τ is proportional to the curvature of the contour and the value of the approximation step Δl. Due to the fact that the section (P_1P_2) is small, the curvature for it can be considered the same at all points. Therefore, the correction angle τ (rad.) is represented as follows:

$$\tau = m\Delta l\rho , \tag{10}$$

where m – coefficient of proportionality;

Δl is the step between reference points, mm;

ρ – the average value of the curvature of the contour $Y = f(X)$ in the section Δl, is determined at the point P_0 according to the following dependence [6]:

$$\rho = \frac{Y''(X)}{(1 + (Y')^2)^{3/2}} .$$

The sign of curvature ρ in expression (10) will depend on the shape of the contour: for a convex $\rho < 0$, for a concave $\rho > 0$. Therefore, the expression for determining the angle γ when processing concave and convex contours has the general form:

$$\gamma = \alpha + \tau . \tag{11}$$

The error will be compensated if the motion trajectory is along a chord, that is, the ratio of feed rates should be such that the direction $\vec{S}_k$ vector coincides with the chord. This condition can be written:

$$\frac{S_Y}{S_X} = \text{tg}\gamma = \text{tg}(\alpha + \gamma) ,$$

or after trigonometric transformations:

$$\frac{S_Y}{S_X} = \frac{\text{tg}\alpha + \text{tg}\gamma}{1 - \text{tg}\alpha \cdot \text{tg}\gamma} .$$

Considering that $\text{tg}\,\alpha = f'(X)$, $\text{tg}\,\tau \approx \tau$ (because the angle τ is small), we get:

$$\frac{S_Y}{S_X} = \frac{f'(X) + \tau}{1 - \tau \cdot f'(X)} . \tag{12}$$

Let's designate the corrected feed ratio defined by expression (12) through K_1. Then the corrected ratio of feed rates, which provides compensation for errors in the deviation of reference points from the nominal profile $Y = f(X)$, can be written:

$$K_1 = \frac{S_Y}{S_X} = \frac{f'(X) + \tau}{1 - \tau \cdot f'(X)} . \tag{13}$$

5.2 Increasing the accuracy of shaping by correcting the ratio of the speeds of the components of the contour feed during the processing of the SCS

To ensure the possibility of correcting the ratio of feeds when machining parts with SCS, it is necessary to determine the angles of rotation of the tangent to the trajectory of shaping in three planes: *X0Y*, *X0Z* and *Y0Z* (in the machine coordinate system).

Consider the geometric meaning of the relationship between feeds in accordance with expression (7).

Geometric meaning $\partial Z / \partial X$ is a slope factor numerically equal to the tangent of the slope of the tangent to the X axis (angle α) drawn in the cross section of the surface $Z = \psi(X,Y)$. Plane $Y = Y_0$ parallel to $X0Z$ to point P_0, Fig. 3. Similarly, the geometric meaning of $\partial Z / \partial Y$ is an angular coefficient numerically equal to the tangent of the slope of the tangent to the Y axis (angle β) drawn in the cross section of the surface $Z = \psi(X,Y)$ by the plane $X = X_0$ parallel to $Y0Z$, at the point P_0, Fig. 4.
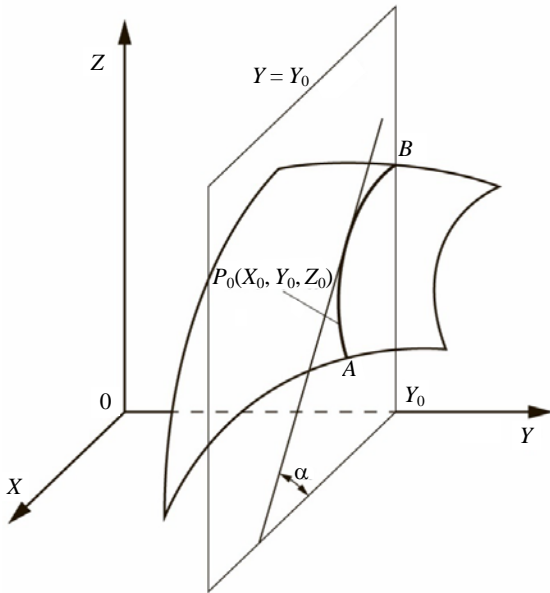

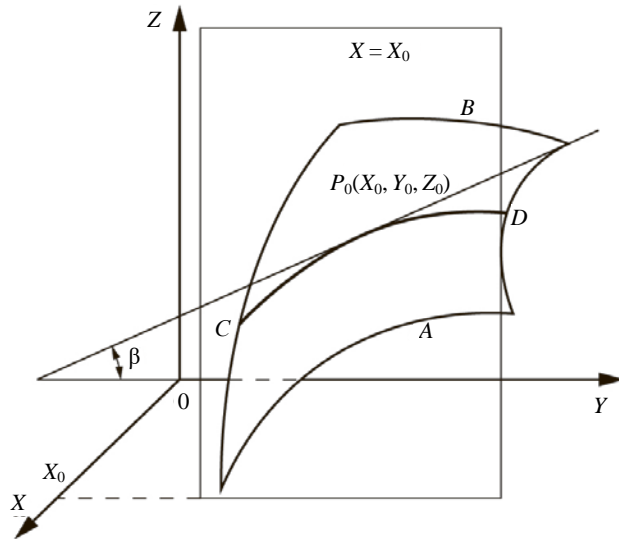**Fig. 3.** Scheme for determining the correction angle in the $X0Z$ plane of the machine

Fig. 4. Scheme for determining the correction angle in the $Y0Z$ plane of the machine

Two tangents to one point $P_0(X_0, Y_0, Z_0)$ of the machined surface $Z = \psi(X,Y)$ determine the tangent plane in which the line of action of the contour feed lies.

To reduce the error of deviation from the shape of the surface at reference points, it is necessary (by analogy with a plane problem) to turn the tangent plane in two planes *(X0Z, Y0Z)* at certain angles. As a result of the additional rotation, the tangent plane should take the position of the cutting plane. The shape of the treated surface and the approximation step Δl determines the angles of correction (adjustment). The value (angle) of the correction is determined while simultaneously solving problems of finding the angles of rotation in the $X0Z$ $Y0Z$ planes.

To compensate for the error in the deviation of reference points from the nominal profile, it is necessary to correct the values $\partial Z / \partial X$ and $\partial Z / \partial Y$.

To determine the value of the correction angles, consider the sections of the machined surface at the point of processing by the $X = X_0$ and $Y = Y_0$ planes Fig. 5, *a*. The section of the processed surface $Z = \psi(X, Y)$, by the plane $Y = Y_0$ represents a flat curve $Z = \psi(X, Y_0)$. The curvature of a flat curve is determined by:

$$\rho_{X0Z} = \frac{Z_X''}{(1 + (Z_X')^2)^{3/2}}.$$

The correction angle is: $\tau_{X0Z} = m \cdot \Delta l \cdot \rho_{X0Z}$.

The corrected value of the coefficient $\partial Z/\partial X$ in expression (7) can be written:

$$K_{X0Z} = \frac{\left(\dfrac{\partial Z}{\partial X}\right)_{Y=Y_0} - \tau_{XOZ}}{1 - \tau_{X0Z}\left(\dfrac{\partial Z}{\partial X}\right)_{Y=Y_0}}. \tag{14}$$
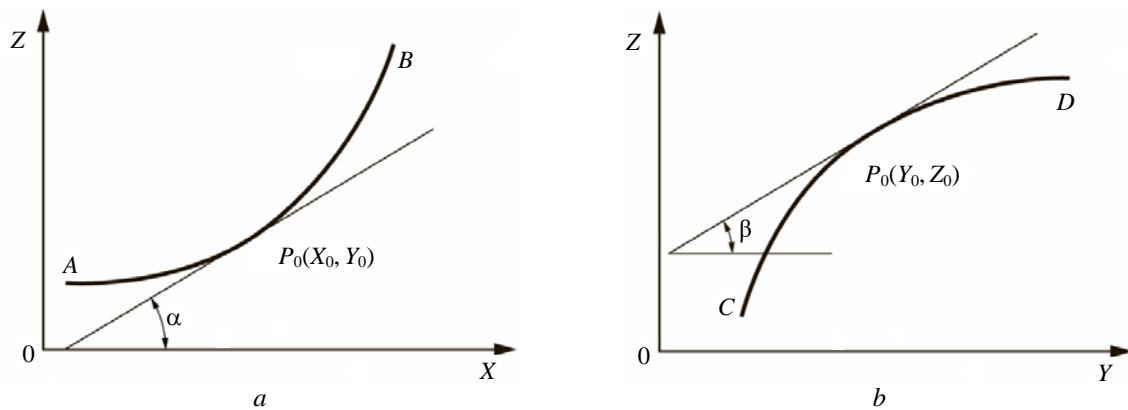
Fig. 5. Section of the surface $Z = f(X, Y)$ by planes: $a - Y = Y_0$; $b - X = X_0$

Similarly, the section of the machined surface $Z = \psi(X, Y)$ by the plane $X = X_0$ represents a plane curve $Z = \psi(X_0, Y)$. The curvature of a flat curve is determined (Fig. 5, b):

$$\rho_{Y0Z} = \frac{Z''_Y}{(1 + (Z'_Y)^2)^{3/2}}.$$

The correction angle is: $\tau_{X0Z} = m \cdot \Delta l \cdot \rho_{XOZ}$.

The corrected value of the coefficient $\partial Z / \partial X$ in expression (7) can be written:

$$K_{Y0Z} = \frac{\left(\dfrac{\partial Z}{\partial Y}\right)_{X=X_0} - \tau_{Y0Z}}{1 - \tau_{Y0Z}\left(\dfrac{\partial Z}{\partial X}\right)_{X=X_0}}. \tag{15}$$

The correction of the coefficient $\partial Y / \partial X$ in the $X0Y$ plane is considered in 4 and, by analogy with (14) and (15), can be written:

$$K_{X0Y} = \frac{\dfrac{dY}{dX} - \tau_{X0Y}}{1 - \tau_{X0Y}\dfrac{dY}{dX}}. \tag{16}$$

Thus, taking into account the correction carried out in the case of a discrete change in the ratios of feeds S_X, S_Y, S_Z, in order to compensate for the error in the deviation of reference points from the nominal surface, expression (7) can be written:

$$S_Z = (K_{XOZ} + K_{YOZ}K_{XOY})S_X, \tag{17}$$

where $S_Y = S_X K_{X0Y}$.

Let us denote the expression in brackets that determines the corrected feed ratio S_Z / S_X through K_2; $K_2 = (K_{X0Z} + K_{Y0Z}K_{X0Y})$.

The ratio of feeds at the reference points can be written:

$$\begin{cases} S_Y = K_1 S_X; \\ S_Z = K_2 S_X. \end{cases} \tag{18}$$

The corrected ratio of feeds (18) provides compensation for the deviation error at the reference points from the nominal shape of the surface $Z = \psi(X, Y)$, which took place when the surface was shaped in accordance with (8) for the case of a discrete change in feeds. However, due to the replacement of the curved section of the surface between adjacent reference points by a chord, an approximation error occurs. Its value depends on the radius of curvature of the surface and the value of the approximation step (length of the chord). Let us determine the relationship between these quantities. To do this, consider a right-angled triangle $A0D$ (Fig. 6). The leg length (AD) is determined by:

$$|AD| = \sqrt{|A0|^2 - |D0|^2} . \qquad (19)$$

Considering that $|AD| = \dfrac{l}{2}$, $|A0| = R$, $|D0| = R - \delta$ expression (19) takes the form:

$$\frac{l}{2} = \sqrt{R^2 - (R - \delta)^2} , \qquad (20)$$

Fig. 6. Scheme of approximation of the curved section of the contour by a chord

where l – chord length (approximation step);

R is the radius of curvature of the curvilinear section;

δ is the approximation error.

Transforming expression (20), we obtain:

$$l = 2\sqrt{2R\delta - \delta^2} .$$

Due to the fact that the value of δ^2 is much less than $2R\delta$, we can neglect it:

$$l = 2\sqrt{2R\delta} . \qquad (21)$$

The value of the approximation step is determined by the value of the contour feed and the value of the time interval Δt between changes in the components of the contour feed S_X, S_Y, S_Z at two adjacent reference points:

$$l = S_k \Delta t . \qquad (22)$$

Let us equate the right parts of expressions (21) and (22):

$$S_k \Delta t = 2\sqrt{2R\delta} . \qquad (23)$$

Rewriting expression (23) as:

$$S_k = \frac{2\sqrt{2R\delta}}{\Delta t} ,$$

we obtain an expression for determining the limiting contour feed, which provides processing with an approximation error δ.

The value of the approximation error δ can be controlled by changing the contour feed, since the radius of curvature depends on the shape of the workpiece, and the time interval Δt between changes in the ratio of feeds along the machine axes is determined by the speed of the machine control system [10].

Conclusions

1. A functional relationship has been established between the geometric shape of a spatially complex surface and the contour feed, which ensures the shaping of the surface by controlling the ratio of the components of the contour feed. This allows you to automate the solution of the trajectory problem in the development of control programs for CNC machines when setting the SCS of the part in the form of an analytical model.

2. An analysis of the accuracy of shaping of spatially complex surfaces was carried out with a discrete change in the ratio of the components of the contour feed, and a method was developed to reduce the magnitude of the shaping error by correcting the ratio of feeds.

3. To achieve the optimal combination of the main indicators of processing efficiency, an algorithm for calculating the components of the contour feed has been developed, which allows you to stabilize the value of the contour feed by controlling the feed depending on the geometric shape of the part.

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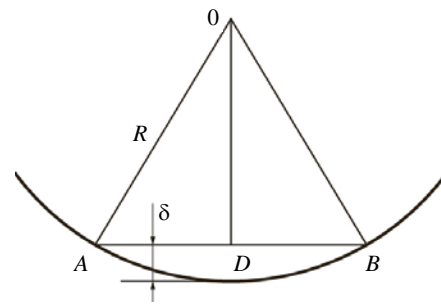
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