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LECTURE NOTES IN ELECTRICAL ENGINEERING THEORY

Topic 010 – Basic Concepts and Laws of Electrical Circuits

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LECTURE 010-01

BASIC CONCEPTS OF ELECTRICAL CIRCUIT THEORY

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- Conclusions

! **You can watch this lecture on the author's YouTube channel "Theoretical Basics of Electrical Engineering" at the link https://youtu.be/sFfdi5M7SKs**

01-1. Electric energy. Its advantages and disadvantages

I want to start with a strange confession for a teacher. I don't know what energy is, what electricity is, and what electrical energy is. No one knows this, so I'm not very embarrassed. Modern science states that energy is a scalar physical quantity that is a general measure of the motion and interaction of all forms of matter. Energy neither arises from nothing nor disappears into nothing; it can only transition from one state to another. The concept of energy unifies all natural phenomena into a whole, providing a general definition of the state of physical bodies and fields.

In turn, it is considered that electrical energy, or electric power, is a form of energy that exists in the form of the potential energy of electric and magnetic fields and the energy of the electric current.

As you can see, all these definitions do not answer the question of what energy is in general and what electrical energy is in particular. Perhaps someday humanity will provide answers to these questions, and most likely, they will be completely different from those presented here.

But until answers to these questions are found, we will move away from them and study how that incomprehensible electrical energy manifests itself and how to predict the consequences of its manifestation. This is the subject of the discipline called "Theory of Electrical Engineering"

First of all, it should be noted that we do not directly consume electrical energy. None of us plugs into an electrical outlet and charges up. We use electrical energy only by transforming it into other forms of energy and for example, converting electricity into heat to make ourselves a morning coffee or warm up by an electric fireplace in cold winter. We convert it into mechanical energy to ride an elevator to a certain floor. We transform it into the energy of sound vibrations while communicating on the phone. And now, if you are reading this text on a computer screen, you are converting electrical energy into light, thanks to which you see something on the screen.

The fact that electrical energy can be easily and with minimal losses transformed into other forms of energy is its significant advantage. Another advantage of electrical energy is that it can be easily and with minimal losses transported over long distances.

And finally, the third advantage of electrical energy is that it can be easily divided into small portions. For example, in the power transmission line from the power station, a very high voltage is received – hundreds of thousands or even millions of volts. However, the voltage in our electrical outlets needs to be significantly lower – 220 volts. This transformation is easily and efficiently performed at distribution substations using transformers.

Advantages of electrical energy:

- **Easily generated**
- **Easily transmitted over long distances**
- **Easily divided and transformed**

However, as you probably already know, in nature, there is nothing that has only advantages. Advantages always come with disadvantages. So, what are the disadvantages of electrical energy?

The main disadvantage of electrical energy is the issue of storage. Electricity is generated, transmitted, and consumed. Problems do not arise when generation and consumption occur simultaneously. However, if we want to generate electricity first but consume it at a later time, then it needs to be stored and preserved. This is where the problem arises. Currently, humanity cannot create storage systems with very large capacities or store electrical energy over an extended period. Typically, electricity is stored in batteries, which have relatively limited capacity. All batteries exhibit the phenomenon of self-discharge, in which the amount of stored electrical energy decreases over time, even when the battery is not connected anywhere. In addition, batteries are heavy and expensive. For example, the traction battery pack for the Tesla Model S electric car, with a capacity of 90 kilowatt-hours, weighs 540 kg, and its dimensions are 210 cm in length, 150 cm in width, and 15 cm in thickness. It can be purchased for

$13,000. One charge of such a battery is sufficient for approximately 500 kilometres of vehicle range, which is roughly six hours of operation.

To study the action of electrical energy in electric circuits, we will rely on concepts such as electric current and electric voltage. You have already learned these concepts in your physics course. Therefore, here we will just recall what they are and provide some definitions familiar to you.

01-2. We reminisce. What is electric voltage?

You have probably heard a lot about electric voltage and likely believe you know what it is. Do you? You're fortunate because, in the world, no one (except you) knows what it is. Indeed, let's turn to the definition of electric voltage provided by Wikipedia. It states that electric voltage is a scalar physical quantity equal to the potential difference between two points in an electric field. Even better, quoting: "Voltage can be represented as the integral along the path L, laid out between points A and B in the electric field from the projection of the field intensity vector onto the tangent to the path L." Clear?

Only one thing is clear. All these definitions are nothing more than an attempt in scientific language to say, "Who knows?" The phenomenon we call "electric voltage" indeed exists. Thanks to this phenomenon, electric charges move, creating what is considered electric current. We will talk about it later. However, modern science cannot explain the physical nature of this phenomenon, just as it cannot explain what an electric field and electric charge are. Therefore, we will not try to explain what voltage is but will focus only on studying how it manifests and what it influences in electric circuits.

To do this, we need to agree on which letter we will use to denote electric voltage in diagrams and formulas. There are two options worldwide - the Latin letters "U" and "V". The most common is the designation with the letter "U", so we will use it. However, keep in mind that in some English literature, you may also encounter the designation with the letter "V".

Electric voltage can be constant, meaning it does not change over time. For such constant voltage, we will use the capital letter "U". However, voltage can be variable, meaning it changes its value over time. In this case, to denote the instantaneous voltage (at a given moment in time), we will use the lowercase letter "u".

! **A constant voltage is denoted by the capital letter "U"**
The instantaneous value of variable voltage over time is denoted by the lowercase letter "u"

The unit of measurement for voltage is the volt (V). One volt is equal to the electrical potential difference that arises in an electric circuit with a constant current of one ampere with a power of one watt. Additionally, one volt is equal to the potential of a

Алессандро Джузеппе Антонио Анастасио Вольта 1745—1827

point in an electric field, where an electric charge of one coulomb has a potential energy of one joule.

This unit is named in honour of the Italian physicist and physiologist Alessandro Volta (1745—1827), who discovered the phenomenon of mutual electrification of dissimilar metals upon contact. He also invented and constructed the first chemical source of a constant electric current, a series of electrical devices (capacitor, electrophorus, electroscope). In 1800, he invented the first electric galvanic battery – the voltaic pile, which you can see in the illustration on the right.

01-3. We reminisce. What is electric current?

You already have an understanding of electric current. Electric current is a phenomenon that arises under the influence of electric voltage. Here is how electric current is defined by Wikipedia: "Electric current is the ordered, directed movement of electrically charged particles in a substance or vacuum." We will return to this definition in sections 01-4 when we discuss what's wrong with electric current. But for now, let's talk about the quantity that characterizes the phenomenon of electric current.

Electric current is characterized by current strength, which is defined as the rate of change of electric charges over time.:

$$i = \frac{dq}{dt},$$

and is measured in amperes (A). This unit is named in honour of the French physicist André-Marie Ampère. The ampere, as a unit of electric current, is one of the seven base units of the International System of Units (SI). Over the years of the existence of this system, the definition of the ampere has been refined several times. The latest change in the definition took place in 2019 and is based on the exact value of the electric charge of the electron (elementary charge). According to this definition, one ampere is the electric current corresponding to the flow of a $\frac{1}{1{,}602176634\cdot10^{-19}}$, that is, 6241509074460762607 elementary charges per second. A more understandable version of this definition states that when the electric current in a conductor is 1 ampere, then in 1 second, a charge equal to 1 coulomb passes through the cross-sectional area of this conductor.

Ампер, Андре-Марі 1775-1836

As inferred from the definitions of electric voltage and electric current, they must have some direction. We will try to understand these directions in the next paragraph.

01-4. We reminisce. Conventional positive directions of voltage and current

As we have already mentioned, the voltage between any two points in an electric circuit is the potential difference of these points. If there is a voltage, it means the potentials of these points are different – in one point, the potential is higher, and in the other, it is lower. If a certain point has a higher potential than another, its potential is considered positive. Accordingly, the potential of the other point is considered negative. A point with a positive potential is denoted by the symbol "+", and a point with a negative potential is denoted by the symbol "−".

For the conventional positive direction of voltage, we will take the direction from the point with a positive potential to the point with a negative potential, as shown in Fig. 01-1 by the blue arrow.

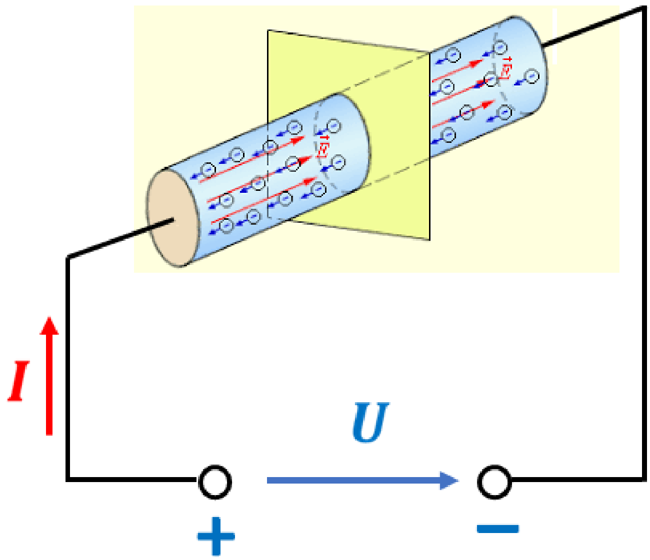


Figure 01-1. Conventional positive directions of voltage and current

Now let's delve into the conventional positive direction of current. Here it gets a bit more interesting. Indeed, as we have already mentioned, current is the directed movement of charged particles. If these particles are negatively charged, such as electrons in metals, then they must move from the negative pole to the positive pole. Recall from physics that like charges repel each other, and opposite charges attract.

However, electric current can arise not only in metals but also in liquids, for example, electrolytes. The charge carriers there are ions, which, in some cases, can have a positive charge. In this case, these positively charged ions should move from the positive pole to the negative pole.

So, what direction of the current will we consider as positive? Even though in the circuits we will study, the charge carriers are electrons, we will consider the direction from a point with a higher potential (positive) to a point with a lower potential (negative) as the conventional positive direction of the current. This positive current direction is shown in Fig. 01-1 by the red arrow.

! The assumed positive direction of voltage and current is considered to be from "+" to "−", meaning from the point with a higher potential to the point with a lower potential

01-5. What's wrong with electric current? Drift velocity of electrons in metals

Remember, not long ago, we promised to delve into the traditional definition of electric current: "Electric current is the ordered, directed movement of charged particles in a substance or vacuum." This definition is repeated so many times that we involuntarily consider it true in the last instance. Let's try to understand this definition in more detail. So, electric current is directed movement. For now, let's not dwell on what exactly is moving. But if there is movement, we can talk about the speed of this movement! Let's talk about it.

Imagine a very long power line, several thousand kilometres long. Experiments show that when you connect a power source to one end of this line, the light bulb at the other end will flash almost instantly, with a delay of a fraction of a second. Experiments show that the speed of propagation of electric current is very high and comparable to the speed of light. Furthermore, in the process of studying electrical engineering, we will learn that it has an order of magnitude of hundreds of thousands of kilometres per second. And we, when we come home in the evening and flick the switch, don't notice any delay between the switch's operation and the appearance of light. However, these same experiments show that electrons, which are the carriers of charge in metals, move at a much lower speed. This speed is called the electron drift velocity and is approximately 0.1 millimetres per second, depending on the metal. This is twenty times slower than the speed at which a snail moves!

And here we have to admit that if electric current were indeed the directed movement of electrically charged particles, then after flicking the switch, the light in our room should only appear after a few hours! Something is not right here! Perhaps electric current cannot be equated with the movement of charged particles!

In the definition of electric current, it involves the movement of electrically charged particles. Let's delve into what electric charge is. Let's turn to modern physics. It states that electric charge is a physical quantity that characterizes the ability of bodies to create electromagnetic fields and participate in electromagnetic interactions. Okay, so the charge is what creates electromagnetic fields. But what is an electromagnetic field? Physics tells us that an electromagnetic field is a field created by moving electric charges.

Strange, isn't it? Charge is what creates the field, and the field is what is created by the charge. It is a comprehensive definition, isn't it?

The fact is that modern science cannot explain what electric current is, what electric charge is, and what an electromagnetic field is. We only know that they exist in the surrounding world and somehow influence the environment. We hope that explanations for all of this are ahead, as science is rapidly advancing.

For now, without understanding what it is, we will study the regularities of how electricity interacts with the surrounding world. We observe and can experimentally investigate this interaction, making calculations. To do this, we need to find out where exactly this mysterious electric current operates.

01-6. Concept of an electric circuit and its elements

The science of "Theory of Electrical Engineering" deals with the calculations of electrical circuits. What is an electric circuit? Wikipedia defines an electric circuit as a combination of interconnected conductors of electrical elements through which an electric current can flow.

! **An electric circuit is a combination of interconnected conductors of electrical elements through which an electric current can flow**

As we can see, an electric circuit consists of individual electrical elements, which, accordingly, are its components. But what is an electrical element?

Wikipedia defines an electrical element as a structurally completed product capable of performing its functions as part of an electric circuit. This definition does not specify what the element is made of. However, an element is a part of an electric circuit. An electric circuit consists of elements. So, we return to quite peculiar definitions that do not define anything!

Nevertheless, understanding what an element is and what a circuit is can be achieved. Let's consider a series of photographs in Fig. 01-2.

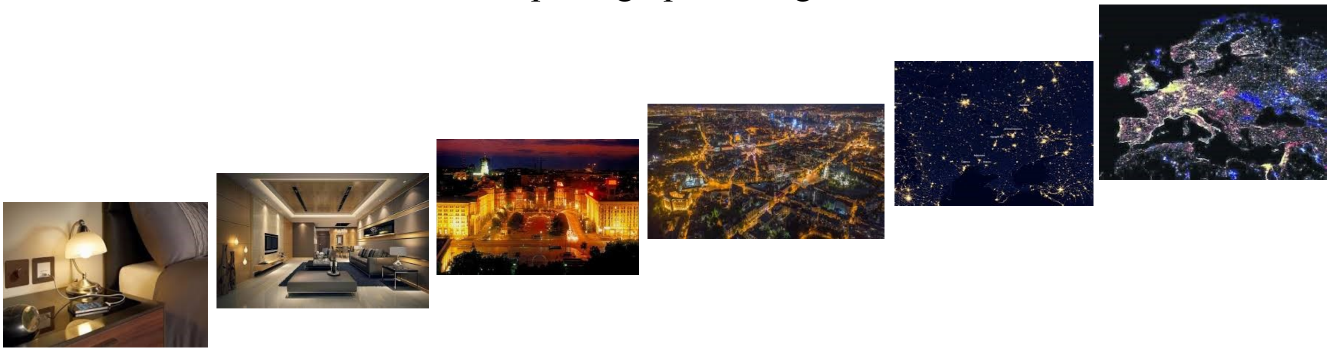


Figure 01-2. Electrical Elements and Electric Circuits

Let's start by examining the bottom-left photo, where an electric light bulb is shown connected to an outlet. Together, the light bulb and the outlet form an electric circuit, while individually, the light bulb and the outlet are its elements.

And now, let's gradually move away from the observation point, encompassing a broader view each time. In the second photo from the left, we see a room in an apartment with several light bulbs and outlets. Thus, at this level, we can discuss the electric circuit of the apartment, with combinations of light bulbs and outlets as its elements.

Moving further away, in the third photo, we see buildings on Independence Square in Kyiv. Each building has many apartments, each of which can be regarded as an element of a larger-scale electric circuit—the circuit of the building.

Continuing our distance, in the fourth photo, we observe a panoramic view of nighttime Kyiv. At this level, we can talk about the electric circuit of the entire city, where individual buildings are its elements.

In the fifth photo, a satellite image of nighttime Ukraine is shown. The bright spots represent individual cities. Here, we can discuss the electric circuit of the entire country (the power system of the country), with cities and villages as its elements.

Finally, the last image depicts nighttime Europe. We can consider individual countries as elements of the electric circuit of Europe. Find our Ukraine in this photo!

Thus, each electric circuit can be an element of a larger circuit. This hierarchy continues, and a larger circuit, if needed, can be viewed as an individual element of an even larger circuit. There is no fundamental difference between a circuit and an element; it depends on the conditions under which we consider the system.

We will begin the study of electric circuits by examining their elements in the next lecture.

01-7. Conclusions

- Electric energy is a form of energy that exists in the form of both the potential energy of an electric field and the energy of an electric current.
- Current and voltage are fundamental phenomena that characterize the operation of any electric circuit.
- Voltage between two points is the difference in electric potentials of these points. We will denote voltage with the letter U for constant voltage or u for time-dependent voltage.
- Electric current is the flow of charged particles moving through an electric conductor. We will denote current with the letter I for constant current or i for time-dependent current.
- Presumably, electric current is likely not just the movement of charges. The concept of current as the movement of charges is just a convenient explanation. In reality, we do not know what electric current truly represents.
- In electrical engineering, it is conventionally chosen that the assumed positive directions of voltage and current are from a point (terminal) with positive potential (+) to a point or terminal with negative potential (−), i.e., from a point with higher potential to a point with lower potential.

– Electric circuits and electric elements are relative concepts. Any electric circuit can be considered as an individual element of a more complex electric circuit.

LECTURE 010-02

PROPERTIES OF ELECTRICAL CIRCUIT ELEMENTS

Lecture Plan

- Summary of the previous lecture
- The properties of electric elements
- The properties of electric elements. The magic of the number 3
- Property 1. Electrical conductivity
- Property 2. Electrical inductance
- Property 3. Electrical capacitance
- Passive and active elements of electrical circuits
- Conclusions

! **You can watch this lecture on the author's YouTube channel "Theoretical Basics of Electrical Engineering" at the link https://youtu.be/h3GpFzTFTAk**

02-1. Summary of the previous lecture

In the previous lecture, we discussed the basic concepts of the theory of electrical circuits. Among these concepts are electrical energy, electric voltage, and electric current. We concluded that the current definitions of these concepts are far from perfect because we don't truly understand what energy, electric charge, and electric current are.

However, this will not hinder us from studying the external manifestations of electrical energy and electric current. We will focus on how they manifest themselves in the surrounding world. The laws governing such manifestations are the subject of the course "Theory of Electrical Engineering."

In the previous lecture, we also provided definitions of what an electric circuit and an element of an electric circuit are. The main conclusion is that there is no fundamental difference between an electric circuit and an element of a circuit; it depends on the scale. For example, in a multi-apartment building, we can talk about the electric circuit of the building, consisting of individual elements - apartments. However, within the city, each building can be considered as one of the elements of the city's electrical network.

Since an electric circuit consists of elements, it can be stated that the behaviour of the entire circuit, i.e., the currents and voltages in it, will be determined by the properties of its elements.

In this lecture, we will answer the question of what properties each element of an electric circuit has. We will also learn that there are two types of electrical elements concerning their relationship to electrical energy - energy consumers and generators.

02-2. Properties of elements of electrical circuits. What is a “property”?

Wikipedia states that the term “Property” refers to a specific characteristic of an object that allows us to distinguish this object from others. If we talk about electrical elements, each of them has properties such as mass, volume, width, length, height, colour, and even smell and taste. Will these characteristics be of interest to us if we study the operation of the element as part of an electrical circuit? Certainly not!

We will be interested only in those properties that are related to the electrical parameters of the circuit, such as current and voltage. We will also be interested in properties that show the interaction of the element with electrical and magnetic fields. How many of such specific electrical properties do you think exist?

The answer will be unexpected. There are only three such properties in the elements.

Let's talk a little about what this number, which is encountered everywhere in our lives, actually represents.

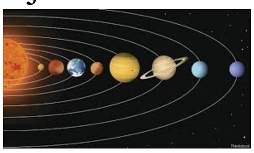
02-3. Properties of elements of electrical circuits. The magic of the number “3”

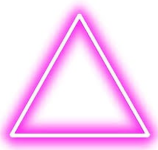
The magic of the number “3” permeates various aspects of culture, religion, and folklore worldwide. Let's provide just a few examples.

The Trinity in religion. Many religious systems incorporate the concept of the Trinity. For example, in Christianity, there is the Holy Trinity (Father, Son, and Holy Spirit). In Indian culture, the triad is found in the tradition of Trimurti (Brahma, Vishnu, and Shiva).

Folklore and fairy tales. The motif of the number three often appears in folk tales and myths. Heroes frequently encounter three trials, three tasks, or three characters, adding drama and symbolism to the narrative.

The number three in art and literature. The principle of triunity permeates art and literature. In many works, a structure with three parts or recurring motifs is used to give a sense of harmony and balance to the piece. In everyday life, we divide a person's age into three parts: childhood, youth, and old age. The structure of power in many democratic systems may be based on the three branches of government: executive, legislative, and judicial.





An interesting manifestation of the number “3” is in science. For instance, we recognize three states of matter (solid, liquid, gas). In mechanics, Newton's three laws govern the behavior of objects. Our Earth is the third planet in distance from the Sun. A triangle is a minimal geometric figure, as a line and point are not geometric shapes.

In the end, the magic of the number “3” permeates many aspects of our culture and science, and its symbolic meaning continues to inspire and captivate the human imagination. In the process of studying the theoretical foundations of electrical engineering, we will encounter the magic of the "three" more than once.

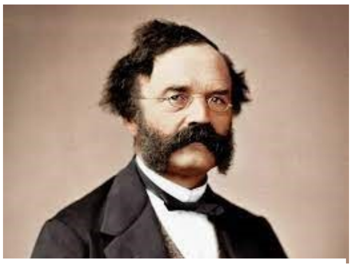
But for now, let's consider the first manifestation of the "three", moving on to the study of three properties of elements of electrical circuits.

02-4. Property 1. Electrical Conductivity

The property of an element to conduct an electric current under the influence of an applied voltage is called electrical conductivity, or simply – conductivity. This property, to a greater or lesser extent, is inherent in every element of an electrical circuit. It is determined by the fact that, when a voltage is applied, the different poles (terminals) of the element have different potentials. As we have discussed, a higher potential is considered positive, and a lower one is negative. Under the influence of the potential difference, the charge carriers within the element begin to move. For example, electrons in metals will be repelled from the negative pole and attracted to the positive one, thus creating an electric current.

! **The property of an element to conduct an electric current under the influence of an applied voltage is called electrical conductivity**

The property of electrical conductivity is characterized by a quantity called "conductance". The greater the conductance, the larger the electric current that occurs in the element at a constant voltage. Conductance is measured in units called "siemens", named after Werner von Siemens, a German engineer, inventor, scientist, industrialist, founder of the Siemens conglomerate, and a public and political figure (December 13, 1816 – December 9, 1892). The term "Siemens" for the unit of electrical conductivity in the International System of Units (SI) was adopted by the XIV General Conference on Weights and Measures in 1971.


Werner von Siemens

It is interesting to note that previously, the unit of conductivity was called 'mho' (the word 'Ohm' spelt backwards) and was represented by the inverted Greek letter omega «℧».

In electrical engineering, conductivity is denoted by the letter "G", and the unit of conductivity, the "siemens", is represented by the capital Latin letter "S". According to the rules of the SI system regarding units named after scientists, the name of the unit "siemens" is written in lowercase, while its symbol is written in uppercase.

Physically, conductivity is a proportionality factor between the current that arises in the element and the voltage applied to that element:

$$i = G \cdot u. \tag{02-1}$$

Often, it is more convenient to use another property of elements, which is closely related to conductivity. This property is the inverse of conductivity, called the resistance of the element, and is denoted as «*R*»:

$$R = \frac{1}{G}.$$

Georg Simon Ohm
1787 – 1854

Resistance is measured in “ohms” and is represented by the Greek letter “Ω”. The unit of resistance is named in honour of the distinguished German physicist Georg Simon Ohm, who dedicated a great deal of effort to studying the mysterious and, to this day, the fundamental phenomenon of electric current. Now, by the way, it is clear why the unit of conductivity was once called “mho”. Simply put, we use the value of an element's resistance much more often than its conductivity. So much so that resistance itself has come to characterize the property of conductivity of an element.

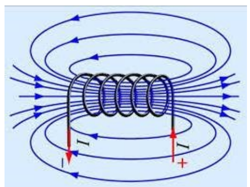
Considering that resistance is the reciprocal of conductivity, formula (02-1) can be rewritten as:

$$u = R \cdot i. \qquad (02\text{-}2)$$

This equation, like equation (02-1), is a fundamental relationship in electrical engineering. You probably know it as Ohm's Law. We will discuss this law in more detail later. But for now, let's move on to the study of the next property of elements in electrical circuits.

02-5. Property 2. Electrical Inductance

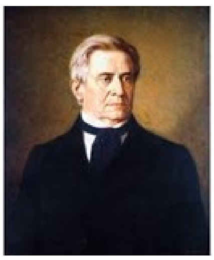
We don't know why, but when an electric current flows through an element, a magnetic field is generated around that element in space. This phenomenon is a result of another property of electrical elements.



The property of an element to create (induce) a magnetic field when an electric current flows through it is called the property of electrical inductance, or simply - inductance.

The property of electrical inductance is characterized by a quantity called “inductance”, denoted by the letter “L”. This quantity is a proportionality factor between the current in the element and the magnitude of the magnetic flux that arises due to this current:

$$\Phi = L \cdot i. \qquad (02\text{-}3)$$



The unit of measurement for inductance in the SI system is the “henry”, denoted by the Latin letter “H”. One henry equals the inductance of an element that induces a magnetic flux of one weber due to a direct current of one ampere flowing through it. This unit is named in honour of the American scientist Joseph Henry. In 1831, he discovered the phenomenon of electromagnetic induction. However, in the same year, the Englishman Michael Faraday independently discovered the same phenomenon but was the first to publish his discovery. Therefore, Michael Faraday is considered the primary discoverer of the phenomenon of electromagnetic induction.

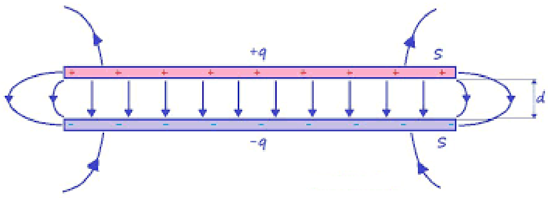
! **The property of an element to create a magnetic field when an electric current flows through it is called the property of electrical inductance**

02-6. Property 3. Electrical Capacitance

The property of an element to accumulate electric charge under the influence of applied voltage is called electrical capacitance. This is the third and final property of elements in electrical circuits. This property is quantitatively characterized by a value known as “capacitance”, which is the proportionality coefficient between the charge accumulated in the element and the voltage across its terminals:

$$q = C \cdot u. \tag{02-4}$$

Capacitance is denoted by the letter “C”. In the SI system, electrical capacitance is measured in “farads” and represented by the Latin letter “F”. An electrical element has a capacitance of one farad if it accumulates a charge of one coulomb under the application of one volt of voltage.



Michael Faraday
1791 – 1867

This unit is named in honour of the eminent English physicist Michael Faraday. As we just mentioned, Michael Faraday discovered electromagnetic induction, which forms the basis of modern industrial electricity production and many of its applications. He created the first model of an electric motor. Among his other discoveries were the first transformer, the chemical action of electricity, the laws of electrolysis, the effect of a magnetic field on light, and diamagnetism. Faraday was the first to predict electromagnetic waves. He also introduced into scientific use terms such as ion, cathode, anode, electrolyte, dielectric, diamagnetism, paramagnetism, and others that we still use today. Faraday is the founder of the study of the electromagnetic field, which was later mathematically formalized and developed by Maxwell. Faraday's main conceptual contribution to the physics of electromagnetic phenomena was the rejection of Newton's action-at-a-distance principle and the introduction of the concept of a physical field as a continuous region of space filled with force lines and interacting with matter.

! **The property of an element to accumulate electric charge under the influence of applied voltage is called electrical capacitance**

The capacitance of one farad is very large. For example, calculations show that a metal sphere with a radius 13 times larger than the radius of the Sun, and accordingly, 1413 times larger than the radius of the Earth, would have such capacitance. Therefore, real existing electrical elements have much smaller electrical capacitance, measured in microfarads ($1\ \mu F = 10^{-6}\ F$), in nano-farads ($1\ nF = 10^{-9}\ F$), and even in picofarads ($1\ pF = 10^{-12}\ F$).

Now we know about the three properties of elements in electrical circuits. These properties are essential for us because we will use them in studying the theoretical foundations of electrical engineering. But before that, we need to understand how elements in electrical circuits are classified based on whether they generate or consume electrical energy.

02-7. Passive and active elements of electrical circuits

All elements that make up an electrical circuit can be divided based on what they do with electrical energy – whether they consume or generate it. According to this characteristic, all elements can be classified into two main groups: passive and active elements (Figure 02-1).

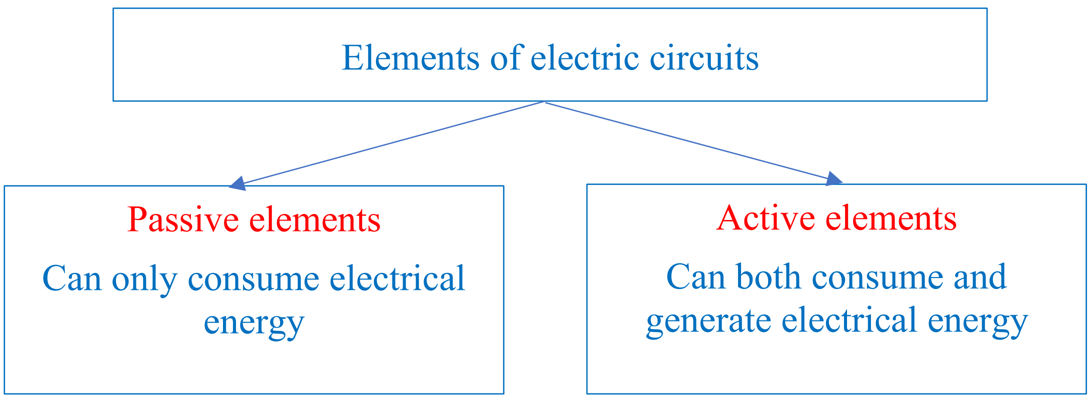


Figure 02-1. Passive and active elements of electrical circuits

As seen from this figure, passive elements are unable to generate electrical energy; they can only consume it. In other words, passive elements are consumers of electrical energy. In contrast, active elements can generate energy. Interestingly, active elements can not only generate energy but, under certain circumstances, also consume it. Therefore, an active element can function in an electrical circuit not only as a generator but also as a consumer.

An example of such an active element, which sometimes can also consume electrical energy, is the battery in our mobile devices. For instance, when we use our smartphone during the day, its battery operates as a generator, supplying power to all its components. However, when we charge the smartphone in the evening, the battery's operating mode changes and it starts to function as a consumer, storing the energy from the electrical network.

How can one distinguish whether an element in an electrical circuit operates as a consumer or a generator? There is a single criterion for determining this: comparing the directions of voltage and current at the terminals of the element. If the voltage and current have the same direction, it unequivocally indicates that the element is a consumer of electrical energy. Conversely, if these directions are opposite, the corresponding element generates electrical energy (Figure 02-2).

a) b)

Figure 02-2. Directions of voltage and current for an energy consumer (a) and an energy generator (b) in an electrical circuit

! **The voltage and current of an energy consumer coincide in direction. The generator has opposite directions of voltage and current at its terminals**

Pay attention that when we say an element is passive, it unequivocally indicates that it is an electrical energy consumer. However, if an element is active, it does not necessarily mean it generates energy. It can both generate and consume it, depending on the operating mode of the electrical circuit. If the voltage and current of the element have opposite directions, then it indeed functions as a generator. But when they coincide in direction, such an active element operates in consumer mode. Remember these signs of a generator and an energy consumer! We will use them repeatedly in the study of the theoretical foundations of electrical engineering.

02-8. Conclusions

- The main electrical properties of elements in electrical circuits are conductivity (resistance), inductance, and capacitance
- Every element in an electrical circuit possesses these three properties
- Elements are categorized as active or passive based on their interaction with electrical energy
- If the current and voltage on an element have the same direction, it is a consumer of electrical energy
- If the current and voltage on an element have different directions, it is a generator of electrical energy
- The presence of these three properties in many elements of an electrical circuit significantly complicates its analysis
- To simplify the analysis, it is necessary to introduce ideal elements of electrical circuits. This will be the topic of our next lecture.

LECTURE 010-03

IDEAL PASSIVE ELEMENTS OF ELECTRICAL CIRCUITS

Lecture plan

- Summary of the previous lecture
- What properties does a real element of an electric circuit have?
- What is idealization
- Ideal passive elements of electric circuits. Resistor
- Ideal passive elements of electric circuits. Inductor
- Ideal passive elements of electric circuits. Capacitor
- Ratio between voltages and currents of ideal passive elements
- Concept of duality in electric circuits
- Conclusions

! **You can watch this lecture on the author's YouTube channel "Theoretical Basics of Electrical Engineering" at the link https://youtu.be/xbyQZAMcBG0**

03-1. Summary of the previous lecture

From the previous lecture, we know that the main electrical properties of elements of electric circuits are conductivity (resistance), inductance and capacitance. We defined these properties and introduced numerical parameters that characterize these properties.

We found out that the elements of electrical circuits are divided into active and passive from the point of view of electrical energy. Passive elements can only consume electrical energy, and the current and voltage on them have the same direction. Active elements can both generate and consume electrical energy. In the case when an element generates energy, the current and voltage on such an element have different directions.

The presence of three properties in many elements of an electric circuit significantly complicates its analysis. Therefore, to simplify the analysis, it is necessary to introduce the concept of ideal elements of electric circuits. This will be the topic of our lecture.

03-2. What properties does a real element of an electric circuit have?

The answer to this question will be very important for us. Let's imagine a real element of an electric circuit - it is several (maybe quite a few) turns of insulated wire wound on a core (Fig. 03-1). What properties does this element have? Let's reflect. First, this coil is wound with real wire that has some electrical resistance. Thus, this element has the property of electrical conductivity. Remember, we said that conductivity and resistance are inverse characteristics.

Secondly, if an electric current acts through this element, a magnetic field will inevitably be created around it. And this, as you already know, is a sign of the property of electrical inductance. Thus, this element has the property of inductance.

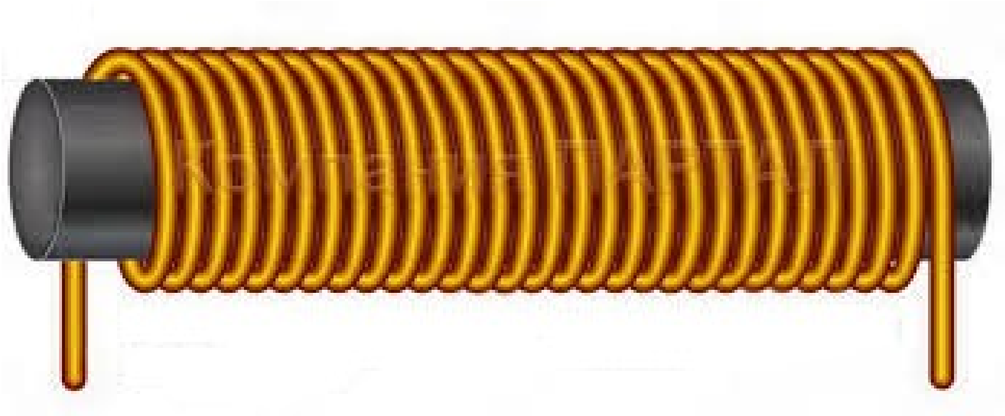


Figure 03-1. A real element of an electric circuit

Well, and thirdly, in the presence of electrical resistance, a potential difference will arise between adjacent turns. And thanks to this difference, an electric charge will accumulate between individual turns of the coil. It is this charge that determines the potential difference. Thus, this element has the property of electrical capacity. Therefore, each real element of an electric circuit has all three properties at the same time - conductivity, inductance and capacity.

! **Each real element of an electric circuit has all three properties - conductivity, inductance and capacity**

The simultaneous presence of three properties in each element will inevitably complicate the analysis of electric circuits consisting of these elements. Therefore, to simplify the further study of electrical circuits, we must also simplify the behaviour of each element. This can be done by introducing the concept of ideal elements of electric circuits.

03-3. What is idealization

Idealization in science means simplifying the real object of research and creating its idealized image, which does not exist in reality but preserves the main qualities of the object with acceptable accuracy. In many cases, idealization is an indispensable tool for studying systems that are too complex to consider in their entirety.

Practically any scientific model, including those used in physics, chemistry, mathematics, etc., contains one or another idealization. In the philosophy of science, two types of idealization are distinguished.

Aristotelian idealization, which is also called minimal, means that we limit ourselves to considering those properties of the object under study, which, in our opinion, significantly affect the problem under consideration, and ignore other properties. An example is the classical model of celestial mechanics, which defines the position of a celestial body as a function of time and ignores all other properties.

Galilean idealization differs in that it allows deliberate distortions of the physical picture - for example, physicists consider point masses that do not exist in nature or bodies that move without friction.

It should be noted that idealization is a very powerful mechanism for studying nature, but at the same time, it is also a very dangerous mechanism. Careless use of idealization can lead to the fact that we lose some important properties of the object, without which it is impossible to build a true model of it.

When idealizing electrical elements, we perform an Aristotelian idealization, assuming that an ideal element has only one of three properties - either only conductivity, only inductance, or only capacitance. At the same time, we assume that the other two properties are completely absent in such an ideal element. It is clear that according to the number of properties of electric elements, we will have three ideal elements of electric circuits. We talked about the magic of the number three in the previous lecture. There we also promised that when studying electrical engineering, we will come across this number more than once. Here is another such meeting.

Let us consider in detail the ideal elements of electric circuits.

03-4. Ideal passive elements of electric circuits. Resistor

An ideal element that has only the property of conduction (resistance), but has no inductance and no capacitance, is called a resistor.

The conventional graphic designation of the resistor on electrical diagrams is shown in Fig. 03-2. The resistor in the diagrams is indicated by the letter «*R*». If the scheme contains several resistors, they are numbered 1, 2, 3, and so on. The number of the resistor, according to DRSTU, is placed next to its designation, as shown in fig. 03-2.

Figure 03-2. Conventional graphic designation of a resistor

! An ideal element of an electric circuit, which has only conductivity, but no inductance and no capacitance, is called a resistor

It is necessary to distinguish between a resistor as an element of an electric circuit, and the resistance of a resistor as a parameter of this element. When we want to mark the resistor with the number 1 on the diagram, we write "R1". And when we want to say that the resistance of this resistor is equal to, let's say ten Ohms, then we will write it as «$R_1 = 10\Omega$». Please note that the resistor number is written as a subscript.

When depicting electrical circuits in our course, we will, for simplicity, use the designation of the resistor number as a subscript.

03-5. Ideal passive elements of electric circuits. Inductor

An ideal element that has only the property of inductance, but no resistance and no capacitance, is called an inductor. On the diagrams, the inductor is marked as shown in Fig. 03-3.

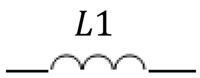


Figure 03-3. Conventional graphic designation of the inductor

! An ideal element of an electric circuit, which has only inductance, but no resistance and no capacitance, is called an inductor

As in the case of a resistor, when we mean an inductor as an element of an electric circuit, its number is written as «$L1$», and when we mean the inductance of this inductor, we will write «L_1».

Note that by definition, an inductor has no resistance, meaning its electrical resistance is zero. This is a very important point to which we will return later.

03-6. Ideal elements of electric circuits. Capacitor

An ideal element of an electric circuit, which has only the property of capacity, but does not have conductivity and inductance, is called a capacitor. In the diagrams, the capacitor is marked as shown in Fig. 03-4.

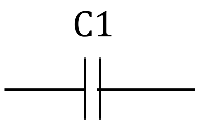


Figure 03-4. Conventional graphic designation of a capacitor

! An ideal element of an electric circuit, which has only capacitance, but no conductance or inductance, is called a capacitor

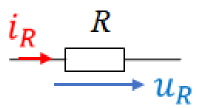
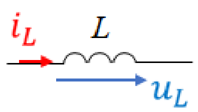
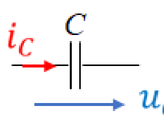
As in the case of a resistor, when we mean a capacitor, as an element of an electric circuit, we will write " $C1$", and when we mean the capacity of this capacitor, we will write " C_1".

Note also that the definition of a capacitor states that it has no electrical conductivity, meaning that the conductivity of a capacitor is zero. But, as we remember, conductivity and resistance are mutually inverse quantities. That is, if the conductivity is zero, then the resistance in this case is infinitely large. Thus, the capacitor has an infinitely large electrical resistance. This is also a very important fact, to which we will return later. Thus, we have just introduced the concept of three ideal passive elements of electrical circuits: a resistor, an inductor, and a capacitor. Such an idealization will allow us to simplify the analysis of electric circuits. After all, for each element of the circle, we will deal with only one property, and not with all three at once. Now it's time to figure out how these ideal elements behave in an electric circuit. We will find out by establishing the relationship between the currents and voltages of each of the three ideal elements.

03-7. The ratio between voltages and currents of ideal passive elements

When an electric current is applied to an ideal element, a certain voltage appears on its clamps. This tension does not arise by itself. The law of change of voltage over time is very closely related to the law of change of current of this element over time. The ratio between currents and voltages of ideal elements of electric circuits is given in Table 03-1.

Table 03-1. The ratio between voltages and currents of ideal elements

Element	$u = f(i)$	$i = f(u)$
Resistor 	$u_R = R \cdot i_R$	$i_R = G \cdot u_R$
Inductor 	$u_L = L \cdot \frac{di_L}{dt}$	$i_L = \frac{1}{L} \cdot \int u_L dt$
Capacitor 	$u_C = \frac{1}{C} \cdot \int i_c dt$	$i_C = C \cdot \frac{du_c}{dt}$

Let's look carefully at this table. The first column shows the designation of each of the three ideal elements and indicates the directions of their currents and voltages. Note that the current and voltage directions on each element are the same. As you remember, this is an indication that the corresponding element consumes electrical energy, that is, it is a passive element.

The second column for each element contains formulas that allow you to calculate the voltage on the element if the law of change of its current is known. After all, in small letters u and i, as we also remember, indicate the instantaneous values of voltages and currents, which change over time according to some law. In the last, third column of the table, there are inverse formulas that allow you to find the current of the element, if the voltage across it is known.

The relationship between the voltage and current of a resistor is already known to you by Ohm's law - the voltage across a resistor is directly proportional to its current. The proportionality factor is the resistance of the resistor. In the inverse ratio, between current and voltage, conductivity is the inverse of resistance as a proportionality factor.

However, for the inductor and the capacitor, the ratios are much more complicated. The voltage on the inductor is directly proportional not to the value of its current, but to the time derivative of the current, that is, the rate of change of the current over time. The proportionality factor between the voltage of the inductor and the rate of change of the current is the inductance of this inductor.

The same can be said for the ratio between the capacitor current and its voltage. Only now the capacity of this capacitor acts as the proportionality factor between the capacitor current and the rate of change of its voltage. The inverse formulas for the inductor and the capacitor contain, as expected, integrals.

Remember this table! We will often use it while studying the theoretical foundations of electrical engineering.

03-8. Conclusions

- All three properties, i.e. conductivity, inductance and capacitance, are always present to some extent for every element of an electric circuit. There are no elements in nature that do not have one or two of these properties
- To simplify the analysis of electrical circuits, idealization is used
- An ideal element of an electric circuit has only one of three properties – either conductivity, or only inductance, or only capacitance
- There are no perfect elements in nature!
- An ideal element that has only conductivity (resistance) but no inductance or capacitance is called a resistor
- An ideal element that has only inductance but no resistance or capacitance is called an inductor.
- An ideal element that has only capacitance but no conductance or inductance is called a capacitor
- There are certain dependencies between currents and voltages of ideal elements. These dependencies must be remembered!

– Electric circuits are characterized by the phenomenon of duality. In electrical engineering, there are dual concepts, dual quantities and dual elements.
– If we introduced ideal passive elements, then we must also idealize the active elements of electric circuits. This will be the topic of our next lecture.

LECTURE 010-04

IDEAL ACTIVE ELEMENTS OF ELECTRICAL CIRCUITS

Lecture plan

– Summary of the previous lecture
– Types of real active elements
– Properties of real active elements
– The first ideal active element. Voltage source
– The second ideal active element. Current source
– Equivalent schemes of real active elements
– Conclusions

! **You can watch this lecture on the author's YouTube channel "Theoretical Basics of Electrical Engineering" at the link https://youtu.be/X5PFN3_h_YU**

04-1. Summary of the previous lecture

In the previous lecture, we introduced the concept of ideal passive elements of electric circuits, as elements that have only one of three properties: either only conductivity, only inductance, or only capacitance. Accordingly, we have introduced three ideal elements of electrical circuits: a resistor, which has only conductance, an inductor, which is a carrier of only inductance, and a capacitor, which has only capacitance.

We also learned that there are certain relationships between currents and voltages of ideal elements of electric circuits, which we recorded in the form of a table. We will often use these relationships in the further study of the theory of electric circuits, so they should always be remembered.

We also said that after idealizing the passive elements of electric circuits, we must introduce the concept of ideal active elements. Our lecture will be devoted to this issue.

04-2. Types of real active elements of electric circuits

As we already know, an active element of an electric circuit is an element that can produce electrical energy. And we are well aware of such elements. There are many of them!

For example, these are elements that produce electrical energy using chemical reactions for this. These are ordinary batteries and accumulators. Batteries are disposable sources - once the battery is discharged, it is

impossible to restore it. But the batteries can be charged. In the process of charging the battery, electrical energy is transformed into chemical energy, and then, during operation, it can be converted back into electrical energy.

In recent years, solar power plants have become very common. But, according to the Eco company Green Energy, global demand for solar energy grew by 40 percent in 2022 and is projected to grow by another 30 percent in 2023. A solar power plant is an active element of an electrical circuit in which light energy is converted into electrical energy.

The most widespread today are sources that convert mechanical energy to produce electrical energy. There are many of them - from mechanical flashlights to nuclear power plants. After all, at a nuclear power plant, the heat produced by a nuclear reactor is first transformed into the mechanical energy of heated steam, which drives a dynamo machine. Well, this dynamo machine already produces electrical energy.

You might ask, why aren't we talking about wind farms? And because they also convert the mechanical energy of the wind into electrical energy.

Despite the great variety of active elements, we can notice one property in them that is inherent in all real active elements without exception. We will now turn to the study of this property.

04-3. Properties of real active elements

Consider the current-voltage characteristic of some active element, for example, a galvanic battery. To obtain the current-current characteristic, it is necessary to assemble the following scheme (Fig. 04-1):

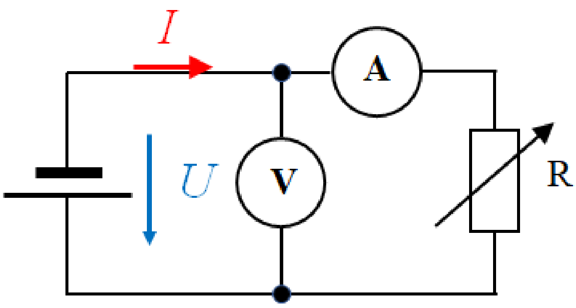


Figure 04-1. Scheme for removing the current-voltage characteristic

The scheme, as you can see, is very simple. A load is connected to the battery – a resistor R, the resistance of which can change. This fact is shown in the diagram by an arrow that crosses out the conventional graphical representation of the resistor. By changing the resistance, we change the current through the battery. The amount of current is controlled by an ammeter, and the battery voltage is controlled by a voltmeter. The current-voltage characteristic of such an active element is shown in Fig. 04-2.

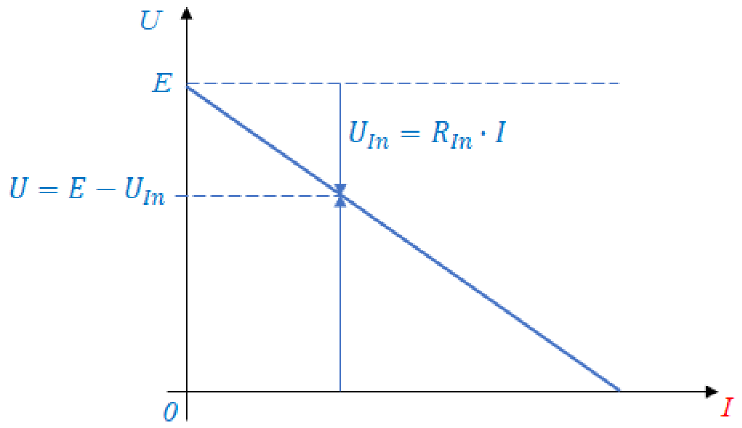


Figure 04-2. Current-Voltage characteristics of a real active element

On this graph, the current is plotted on the abscissa axis, and the voltage on the element is plotted on the ordinate axis. As can be seen from this graph, when the current through the active element increases, the voltage on it decreases, that is, the current-voltage characteristic has a decreasing character.

This behavior of the current-voltage characteristic is because any real active element has an internal resistance R_{In}. If the current is zero, then the voltage on the element is maximum and is equal to the electromotive force of this element (E). When the current through the element increases, the voltage on this internal resistance increases too, which can be calculated according to Ohm's law:

$$U_{In} = R_{In} \cdot I.$$

This voltage is harmful because it reduces the external voltage on the element terminals:

$$U = E - U_{In}.$$

This is exactly what we see in Fig. 04-2.

Thus, from the analysis of the current-voltage characteristic, we can conclude that the voltage at the terminals of a real active element depends on its current.

! **The voltage at the terminals of a real active element depends on its current**

This property is inherent in any active element regardless of its nature - from a watch battery to a nuclear power plant.

Let us now begin to idealize the active elements.

04-4. The first ideal active element. Voltage source

An ideal element, the voltage at the terminals which does not depend on its current, is called a voltage source.

It follows from this definition that the voltage-current characteristic of the voltage source should have the form as shown in Fig. 04-3:

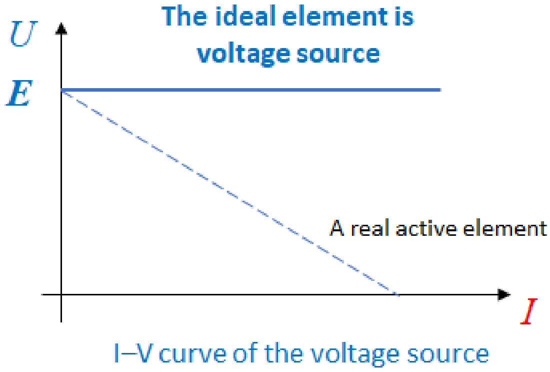


Figure 04-3. Current-Voltage characteristics of the voltage source

Indeed, by definition, the voltage at the terminals of the voltage source remains unchanged and is always equal to its electromotive force. This can only be if the internal resistance R_{In} of this source is zero. Indeed, in this case, the voltage drop inside the source will always be zero, regardless of the amount of current. Accordingly, all the voltage generated by the source (and it is equal to its electromotive force) will enter the electric circuit without losses.

! **The voltage at the terminals of the voltage source does not depend on its current. The internal resistance of the voltage source is zero**

In turn, the current of such a source can be any, both in terms of magnitude and direction. Its current does not depend on the voltage at the source. This current depends only on factors external to the source, that is, on the circuit to which the current source is connected.

If the voltage of the voltage source (its electromotive force) changes over time, it is indicated by a small letter " e" (Fig. 04-4). In the case of a voltage that is constant in time, a capital letter " E" is used.

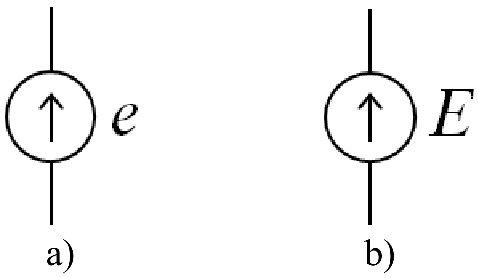


Figure 04-4. Conventional graphic notation of the source of variable (a) and constant (b) voltage over time

The arrow inside the designation of the voltage source points to the clamp that has a higher potential, that is, to the positive pole of the source (Fig. 04-5).

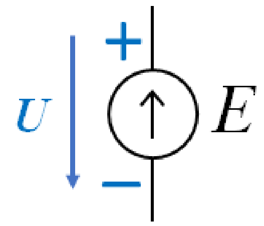


Figure 04-5. The direction of the voltage on the voltage source

Please pay attention to the direction of the electromotive force and the direction of the voltage on the voltage source. In lecture 010-01, we agreed that we would always choose the direction from the positive pole of the element to its negative pole as the conditional positive direction of the voltage. Please look again at Figure 01-1 where this is shown.

Thus, the voltage on the terminals of the voltage source is always directed in the direction opposite to its electromotive force. This is very important to understand and remember because we will meet with voltage sources often.

! The direction of the voltage at the terminals of the voltage source is always opposite to the direction of its electromotive force

Idealization of real active elements in the form of a voltage source is not the only possible option. Now we turn to the study of another ideal active element of electric circuits.

04-5. The second ideal active element. Current source

An ideal element, the current at the terminals which does not depend on its voltage, is called a current source. The current-voltage characteristic of such a source is a straight line perpendicular to the current axis (Fig. 04-6).

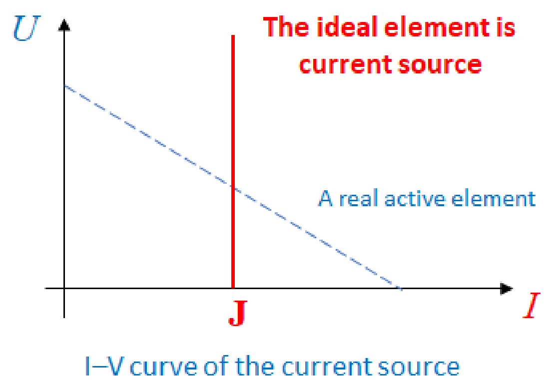


Figure 04-6. Current-Voltage characteristics of the current source

The current passing through the current source is constant and does not depend on its voltage. In turn, we can say that the voltage on such a source also does not depend on its current and can have any magnitude and direction. This voltage depends only on the parameters of the electrical circuit to which the current source is connected.

! An ideal element, the current at the terminals of which does not depend on its voltage, is called a current source

The conventional graphic designation of the current source is shown in Fig. 04-7.

Figure 04-7. Conventional graphic designations of the current source
a) – time-varying current; b) – constant current in time

The direction of the current generated by the current source is shown by the arrows on this notation. However, the voltage on this source can be arbitrary both in magnitude and direction. Both the magnitude and direction of this voltage depend on the electrical circuit to which the current source is connected.

! The voltage at the terminals of the current source can be any and is determined by the parameters of the circuit to which this source is connected

Let's look again at the volt-ampere characteristic of the current source (Fig. 04-6). As you can see, the current of this source remains constant and does not depend on the parameters of the external electrical circuit to which the source is connected. At any resistance of this circuit, the current is constant. This is possible only on the condition that no matter how large this external resistance is, it will still be much smaller compared to the internal resistance of the source. That is, the internal resistance of the current source is infinitely large.

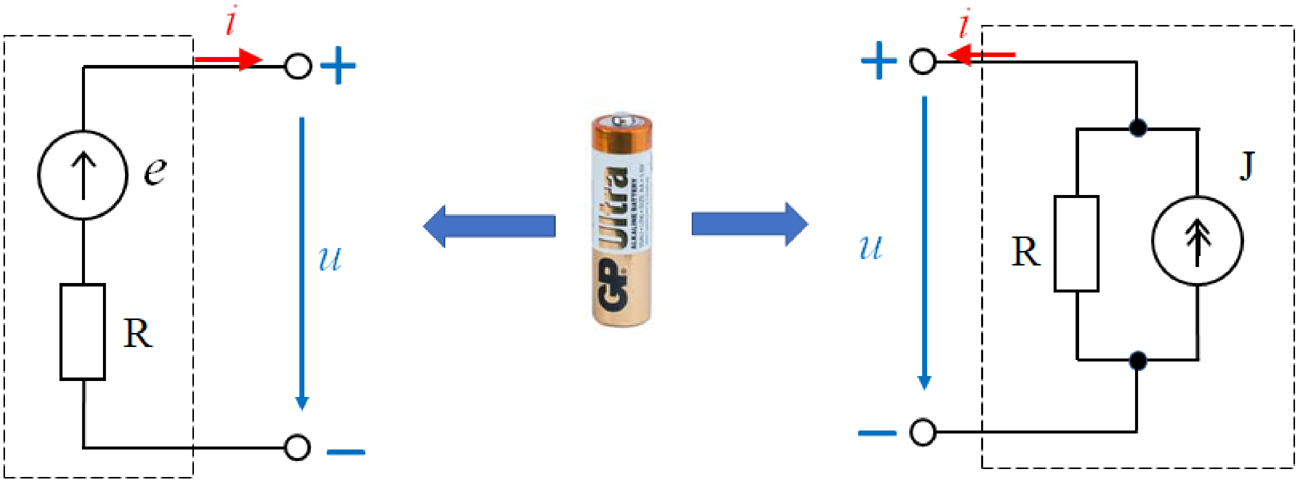
! The internal resistance of the current source is infinitely large

As you can see, from the point of view of internal resistance, a current source is the exact opposite of a voltage source. For a voltage source, the internal resistance is zero, and for a current source, it is infinite. Note that when the resistance is infinite, the conductance, the inverse of the resistance, is zero. That is, the internal conductivity of the current source is zero. This fact will later help us to make a very important fact about voltage source and current source.

Thus, we have idealized a real active element, with some internal resistance of its own, using two ideal active elements. This will allow us to build equivalent schemes for a real active element.

04-6. Equivalent schemes of real active elements

Any real active element of an electric circuit can be equivalently represented by two equivalent schemes - using either a voltage source or a current source (Fig. 04-8).



a) b)

Figure 04-8. Equivalent schemes for a real active element
a) – using a voltage source; b) using a current source

Do you remember? Every real active element has an internal resistance. Therefore, we can model the properties of a real element in two ways.

First, you can use a voltage source with a resistor connected in series with it (Fig. 04-8 a). In this case, the electromotive force of a real active element is simulated by a voltage source, and the internal resistance of a real source is simulated by a series-connected resistor.

The same effect can be achieved by using a current source instead of a voltage source. However, the internal resistance of the current source is infinitely large. Therefore, a resistor simulating the internal resistance of a real source must be connected in parallel to the current source, as shown in Fig. 04-8 b.

Note that one real active element is modelled with two ideal ones, that is, the equivalent scheme is complicated. But, as we will see later, thanks to the ideality of the elements, its analysis will not be difficult.

Both equivalent schemes simulate one real active element. Therefore, we can assume that they are closely related to each other. For example, by knowing the parameters of the circuit elements with a voltage source, we will be able to determine the parameters of the circuit with a current source. We will return to this when we study the equivalent transformations of electric circuits.

04-7. Conclusions

- Any real active element in an electric circuit has an internal resistance
- The volt-ampere characteristic of a real active element has a decreasing character: as the current through the element increases, the voltage across it decreases
- An ideal element, the voltage on which does not depend on the current, is called a voltage source
- An ideal element whose current does not depend on voltage is called a current source
- The internal resistance of the voltage source is zero, and the current source is infinitely large
- Any real active element of an electrical circuit can be represented by an equivalent circuit that contains either a voltage source or a current source.

Now we know almost everything about the elements of electric circuits and their properties. However, electric circuits and their elements have another mysterious feature, which we will get to know in the next lecture.

LECTURE 010-05

THE CONCEPT OF DUALITY IN ELECTRICAL CIRCUITS

Lecture plan

- Summary of the previous lecture
- A new look at the relationship between currents and voltages of passive elements
- What is duality?
- Duality of ideal passive elements of electric circuits
- Duality of ideal active elements of electric circuits
- Conclusions

! **You can watch this lecture on the author's YouTube channel "Theoretical Basics of Electrical Engineering" at the link https://youtu.be/QgGNgSatYYk**

05-1. Summary of the previous lecture

In the previous lecture, we found out that any real active element of an electric circuit has an internal resistance. It is the presence of this resistance that causes the decreasing character: as the current through the element increases, the voltage on it decreases. This resistance is harmful, since part of the electrical energy produced by the active element does not reach the external circuit, but is lost inside this element.

We introduced the concept of two ideal active elements of electric circuits precisely based on the value of internal resistance. We called an element whose internal resistance is zero a voltage source. The voltage on it does not depend on the strength of the current through this element. If the internal resistance is directed to infinity, we get a current source. The current of such a source does not depend on the voltage on it.

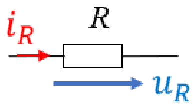
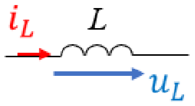
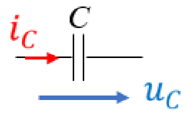
Any real active element of an electrical circuit can be represented by an equivalent circuit that contains either a voltage source or a current source.

But all electric circuits have one very interesting feature, which we will talk about in this lecture.

05-2. A new look at the relationship between currents and voltages of passive elements

In lecture 010-03, studying the features of passive elements of electric circuits, we built a table of correlations between currents and voltages of passive elements. It is very important, so we will repeat it.

The ratio between voltages and currents of ideal elements

Елемент	$u = f(i)$	$i = f(u)$
Резистор 	$u_R = R \cdot i_R$	$i_R = G \cdot u_R$
Котушка 	$u_L = L \cdot \frac{di_L}{dt}$	$i_L = \frac{1}{L} \cdot \int u_L dt$
Конденсатор 	$u_C = \frac{1}{C} \cdot \int i_C dt$	$i_C = C \cdot \frac{du_C}{dt}$

Let's look carefully at the first row of our table. It contains two formulas:

$$u_R = R \cdot i_R \qquad (05-1)$$

and

$$i_R = G \cdot u_R. \qquad (05-2)$$

First, it should be noted that each of these formulas is true. Both of them set the ratio inherent in the resistor. Second, look at how similar they are in structure! We can replace voltage in the first formula u_R on current i_R, resistance R replace by conductivity G, and we get the second formula! Pay attention, we took one true formula, replaced it in a certain way the quantities included in it, and got another true formula as a result.

Let's take the relationship between the currents and voltages of the inductor and the capacitor. For the inductor we have the formula

$$u_L = L\frac{di_L}{dt} \qquad (05-3)$$

and for the capacitor -

$$i_C = C\frac{du_C}{dt}. \qquad (05-4)$$

Again, if we replace in one correct formula, for example, in the formula for the voltage on the inductor, we replace the voltage with the current, and instead of the inductance of the inductor we put the capacitance of the capacitor, then we get another formula that correctly describes the interdependence between the voltage and the current of the capacitor.

Is this a coincidence? It turns out not! You and I have just lifted the veil of mystery from one incomprehensible aspect of our lives - from the phenomenon of duality.

05-3. What is duality?

Duality is an important aspect of our world that manifests itself in many areas of our lives. It indicates the existence of two opposite, but interdependent aspects that form a single whole.

In physics, duality can manifest itself in the form of a wave and at the same time corpuscular nature of matter, where particles can behave like waves, and vice versa. This phenomenon is called "corpuscular-wave dualism". In mathematics, this can appear in the relationship between formulas or operations, where one can be considered as dual to the other.

Duality also exists in the spheres of society and relationships. It can manifest itself in different points of view and approaches that exist in parallel, interact with each other, but together create the multifaceted nature of our society.

A typical example of this is the two-party system. So, for example, the Democratic and Republican parties traditionally play a key role in the political life of the USA, and the Labor and Conservative parties historically dominate the political life of Great Britain. Note that the political systems of the United States and Great Britain have been stable for hundreds of years!

The philosophy sees duality as an important aspect of human existence, where opposing aspects such as body and mind, free will and necessity, coexist and interact.

Thus, duality defines our perception of the world through interconnections and interdependencies, creating a complex image of our existence. This concept expands our understanding of reality, contributing to the discovery of new depths and perspectives in the study and interaction with the surrounding world. This also applies to electrical engineering.

In science, we will call such mathematical expressions, which pass from one to another and preserve the correct description of certain processes, dual expressions, and these processes - dual processes. We will call the quantities included in these expressions dual quantities.

You and I will still use the opportunity that duality gives us - the opportunity to take one true formula (or system of formulas), replace all quantities in it with dual ones, and then get another formula that correctly describes the dual process.

! **Duality makes it possible to take one true formula, replace all quantities in it with dual ones, and then get another formula that correctly describes the dual process.**

05-4. Duality of ideal passive elements of electric circuits

Let's consider formulas (05-1) and (05-2) once again. Replacing voltage with current in the first, and resistance with conductivity, we get the second formula. That is, these formulas are dual. The quantities included in them are also dual - voltage and current, resistance and conductivity.

From formulas (05-3) and (05-4), in addition to the already familiar dual pair "voltage - current", we can obtain another pair of dual values "inductance - capacitance". Elements carrying the properties of inductance and capacity will also be dual. This, as you already know, is a capacitor and an inductor. The processes described by formulas (05-3) and (05-4) are also dual, i.e. the process of the current in the inductor and the process of the voltage on the capacitor.

Let's present pairs of newly found dual concepts and quantities in the form of a table (table 05-1).

Table 05-1. Dual concepts and values for passive elements

Concept (quantity)	Dual concept (quantity)
High-voltage	Current
Resistance	Conductance
Inductance	Capacity
Inductor	Capacitor

This table is not yet complete. And we will continue it right now.

05-5. Duality of ideal active elements of electric circuits

Let's go back to lecture 010-04, where we discussed ideal active elements. There we gave the following definitions for voltage source and current source:

– An ideal element, the voltage on which does not depend on the current, is called a voltage source;
– An ideal element whose current does not depend on voltage is called a current source.

We already know that voltage and current are dual concepts. Let's take the first definition and replace the word "Voltage" with the word "Current" in it, and vice versa - "Current" with "Voltage". See, after this substitution, we have converted the definition of a voltage source to a definition of a current source. That is, we can say that the voltage source and the current source are dual objects. We can also consider the parameters of these objects to be dual - the electromotive force of the voltage source E and the current of the current source J. This allows us to expand Table 05-1 with new data (Table 05-2).

Table 05-2. Dual concepts and quantities in electrical engineering

Concept (quantity)	Dual concept (quantity)
High-voltage	Current
Resistance	Conductance
Inductance	Capacity
Inductor	Capacitor
Voltage source	Current source
EMF of the voltage source ($\boldsymbol{E}$)	Current source current (J)

This table is also not yet complete. We will supplement it in our next lectures.

05-6. Conclusions

1. Duality is a concept characteristic of all objects and phenomena of nature
2. In physics and electrical engineering, there are dual concepts, dual mathematical formulas and dual quantities
3. The relationship between currents and voltages of passive elements is dual
4. Voltage and current, resistance and conductivity, the inductance of an inductor and the capacity of a capacitor are dual quantities
5. A voltage source and a current source are dual concepts, and their parameters - EMF of a voltage source and current of a current source are dual quantities
6. The list of dual concepts and quantities will be continued

In the next lecture, we will learn something new about the topology of electric circuits.

LECTURE 010-06

BASIC TOPOLOGICAL CONCEPTS OF ELECTRIC CIRCUITS

Lecture plan

- Results of the previous lecture
- What is topology
- Topological concepts of electrical circuits: branch, node, mesh
- The concept of independent nodes and meshes
- An example of building a system of independent nodes
- An example of building a system of independent meses
- Conclusions

! You can watch this lecture on the author's YouTube channel "Theoretical Basics of Electrical Engineering" at the link https://youtu.be/cpeysufang8

06-1. Results of the previous lecture

In the previous lecture, we learned about the concept of duality, as inherent in all phenomena and objects of nature. Including, duality also exists in electric circuits. We found out that in physics and electrical engineering, there are dual concepts, dual mathematical formulas (relationships) and dual quantities.

In electric circuits, the mathematical relations between currents and voltages of passive elements are dual.

Voltage and current, resistance and conductance, the inductance of an inductor and the capacitance of a capacitor are dual quantities.

A voltage source and a current source are dual concepts, and their parameters - EMF of a voltage source and current of a current source – are dual quantities. We will continue the list of dual concepts and quantities.

And in this lecture, we should get acquainted with the basic topological concepts of electric circuits, which are the basis for their calculations. But first of all, let's talk about what the science of "topology" is.

06-2. What is topology?

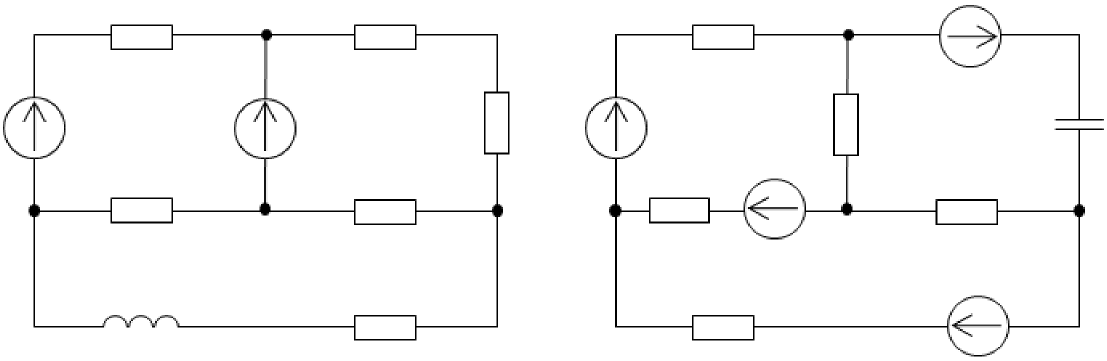
Topology is a branch of mathematics, namely geometry. This science studies the laws of connectivity and mutual location of points, lines, surfaces, bodies and their parts in space, regardless of the sizes and properties of these bodies.

The word "topology" comes from two Greek words τόπος – place and λόγος – word, teaching.

The central concept of topology is homeomorphism, or topological equivalence. Two objects are called homeomorphic if there is a one-to-one continuous mapping of any of them to the other object. This means that we can take an object and change its shape without breaking the surface of the object. Two objects with the original shape and with the shape changed in this way will be topologically equivalent.

Such properties are inherent, for example, in the cow and the sphere, which are shown in these figures. The presentation of this lecture, which you can find on the YouTube channel "Theory of Electrical Engineering" at the link https://youtu.be/cbTDNYgioGQ , shows how these two figures transform into each other. That is, the figure of an animal and the figure of a sphere are topologically equivalent.

In electrical engineering, we will talk about topologically equivalent circuits. Consider, for example, the two schemes shown in fig. 06-1.



a) b)

Figure 06-1. Topologically equivalent electrical circuits

These schemes are different, because they do not repeat each other in terms of composition and location of specific elements. But if you don't take into account which elements are in a certain place in these schemes, you can see that their connection is similar. Let's just look at how the elements are connected, without paying attention to which elements are connected. Moreover, we cannot depict these elements at all, because we are only interested in the way they are connected. Depicting only the lines that show the connection of elements, we get one topological scheme, which is equivalent to scheme a) and scheme b) in Fig. 06-1. Such a topologically equivalent scheme is shown in Fig. 06-2.

The scheme shown in Fig. 06-2 is also called a skeleton circuit diagram. Skeleton diagrams simplify the analysis of electrical circuits.

! The use of skeleton diagrams simplifies the analysis of an electrical circuit

Instead of a detailed display of each component, they represent the basic structure of the system, which allows you to quickly understand the relationships between elements.

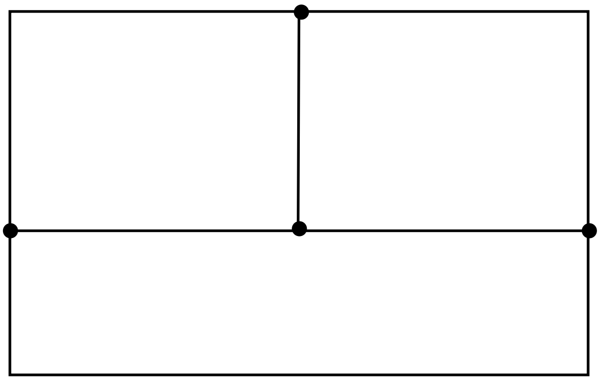


Figure 06-2. The skeleton diagram of circles shown in Fig. 06-1

At the same time, the skeleton circuit diagram allows us to move on to the study of the main topological concepts of electrical circuits - to the concepts of branch, node and mesh.

06-3. Topological concepts of electrical circuits: branch, node, mesh

Let us define these three topological concepts.

We will call a branch the part of an electric circuit located between two of its nodes. In turn, the branches of the scheme are divided into independent and dependent. An independent branch is a branch that does not contain a current source. At the same time, if a current source is present in the branch, we will call such a branch dependent. After all, the current of this branch cannot take any value but completely depends on the current of the current source.

The junction of three or more branches is called a node.

A mesh is a closed path along independent branches of the circuit. The fact that a path is closed is indicated by the fact that it must begin and end at the same point of the diagram. You can pass by the same branch only once. Please note that only independent branches can be part of the mesh!

! **A branch is called independent if it does not contain a current source**
Only independent branches can be part of the mesh

Such definitions of the concepts of a node and a branch may seem a little strange at first glance, but it is intuitively clear from them what a node is and what a branch is. To fully understand what a branch, node and contour are, let's turn to Fig. 06-3, which shows the skeleton diagram of some electric circuits. In this skeleton diagram, nodes are indicated by dots. In each node, the branches included in it have an electrical connection

. When displaying some schemes, the branches of the scheme may intersect, but they are not electrically connected in a circle. In this case, a point is not placed at the point of intersection, which emphasizes the absence of a node at this point.

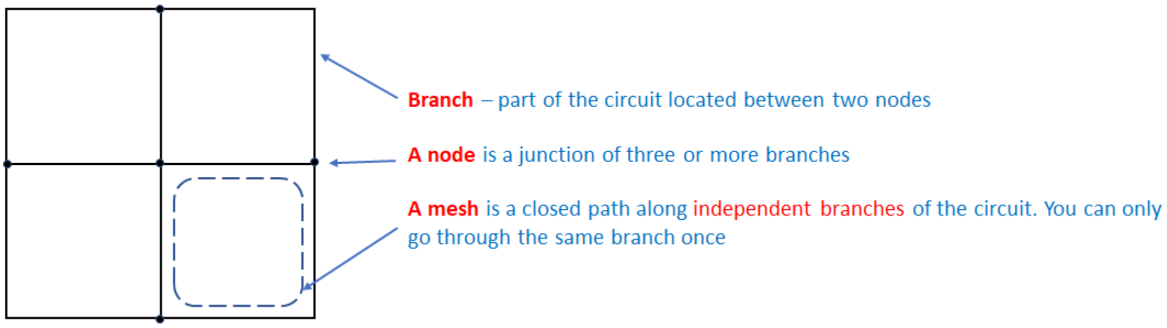


Figure 06-3. Skeleton diagram of a circle

It is not difficult to see that a circle with such a skeleton diagram has five nodes (points in the figure). There are eight branches between these nodes. This is easy to see and calculate. And how many meshes does this scheme have? Here the question is more complicated because we need to see and count all possible closed paths along the branches of our scheme.

Let's start the calculation, assuming that all branches of the circuit are independent, that is, the circuit does not contain current sources. In Fig. 06-4 shows the first four meshes of the scheme in red:

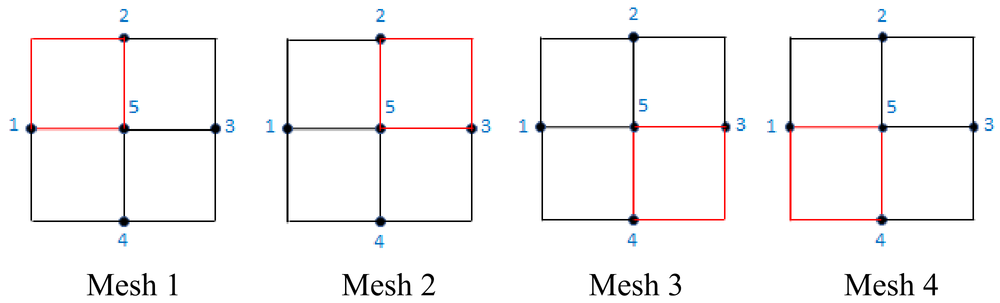


Figure 06-4. The first four meshes of the scheme

Each of these meshes contains three branches. These are the shortest options of a closed path along the branches of this scheme. But there are more options, even more difficult ways. In Fig. 06-5 shows four more meshes, each of which already contains four branches:

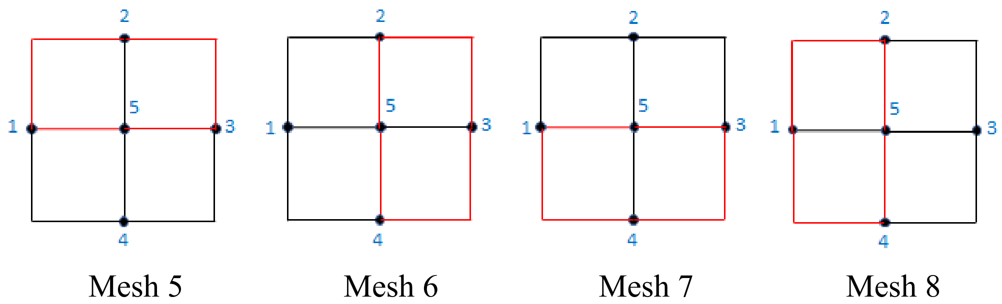


Figure 06-5. The second four meshes of the scheme

In Fig. 06-6 shows another series of four meshes, each of which already contains five branches:

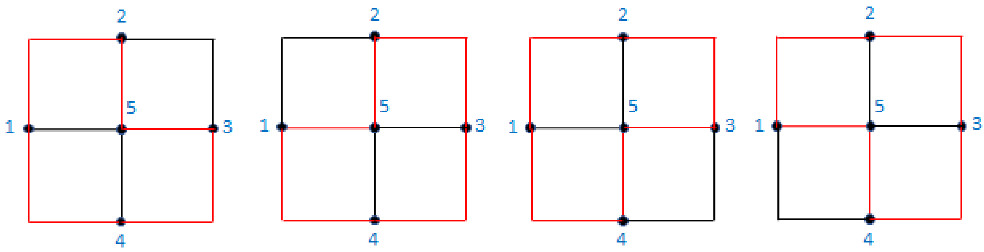
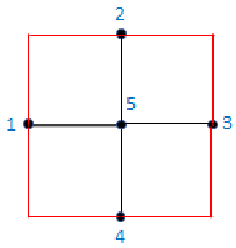


Figure 06-6. The third four meshes of the scheme

And, finally, you can find one more, the last mesh of this scheme, which passes along all four external branches (Fig. 06-7):



Mesh 13

Figure 06-7. The last mesh of the scheme

Thus, a fairly simple scheme has a large number of meshes, which are often quite difficult to find. But in the future, we will not need to do this, because now we will introduce the concepts of independent nodes and independent meshes of the circuit. We will use such independent nodes and meshes in the analysis of electric circuits.

06-4. The concept of independent nodes and meshes

The basic laws of electric circuits, which we will talk about in the next lecture, use the concepts of independent nodes and independent meshes.

An independent node is a node that includes at least one new branch that is not part of other nodes.

A mesh is called independent if it includes at least one new branch that is not part of other meshes.

Notice how similar these definitions are! If in the first of them the word "node" is replaced by the word "mesh", then we will get the second definition. In the previous lecture, we called such definitions dual. We will return to the duality of nodes and meshes, but for now, let's illustrate with the example of our skeleton diagram how you can build a system of independent nodes and a system of independent meshes.

06-5. An example of building a system of independent nodes

The basis for building a system of independent nodes is their definition.

! **An independent node is a node that includes at least one new branch that is not part of other nodes**

Let's consider the construction of a system of independent nodes using the example of the skeleton scheme familiar to us, shown in Fig. 06-3. We will consider this construction in detail and step by step, but at the end, we will learn how this work can be greatly simplified.

We arbitrarily number the nodes on the structural diagram of the circle, as shown in Fig. 6-8.

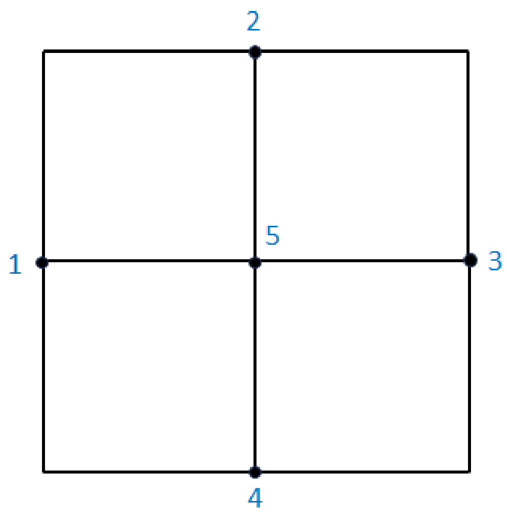


Figure 6-8. Electric circuit nodes

Now we will take each node of the scheme one by one and see if it includes at least one new branch that is not yet included in any of the considered nodes. The results are entered in Table 06-1.

If the next branch under consideration has not yet been included in any of the considered nodes, then we will highlight it in the table in green. If it is already found in the table earlier, we will highlight it in red.

Consider node 1. It includes branches 1-2, 1-5 and 1-4. We enter these data in the first line of Table 06-1. None of these branches go into the nodes previously discussed, because we don't have any such nodes yet! Therefore, all these branches are marked in green in the table. As you can see, node 1 includes three new branches, so this node is independent.

Consider node 2. It includes branches 1-2, 2-3 and 2-5. Branch 1-2 was already included in our table when we considered node 1. It is marked in red in row 2 of Table 06-1. But branches 2-3 and 2-5 are new. Thus, node 2 includes two new branches, and this node is independent.

Table 06-1. A system of independent nodes

Node	Branches	Independent?
1	1-2; 1-5; 1-4	Yes
2	1-2; 2-3; 2-5	Yes
3	2-3; 3-5; 3-4	Yes
4	1-4; 4-5; 3-4	Yes
5	1-5; 2-5; 3-5; 4-5	No

Let's go to node 3. It is formed by branches 2-3, 3-5 and 3-4. Among them, branch 2-3 have already met before, it was part of node 2. Branches 3-5 and 3-4 are new here, so node 3 is also independent.

Consider node 4. It is formed by branches 1-4, 4-5 and 3-4. Two of them, namely branches 1-4 and 3-4, were already included in previous nodes. And branch 4-5 is new, so node 4 is also independent.

Node 5 includes four branches: 1-5, 2-5, 3-5 and 4-5. But all of them were already part of the previously considered nodes. Node 5 contains no new branches, so this node is not independent.

Thus, the electrical circuit, the skeleton diagram of which is shown in Fig. 06-8, has five nodes, four of which are independent. That is, the number of independent nodes in our scheme is one less than the total number of nodes. In graph theory, it is shown that this rule is valid for all schemes.

! **The number of independent nodes in the scheme is always one less than the total number of nodes in it**

Using this rule, you can make it much easier to choose a system of independent nodes. Let's denote the total number of nodes in the scheme as N_{Nd} (from the English "Node"). Then the number of independent nodes N_{INd} (Independent Node) can be calculated as:

$$N_{INd} = N_{Nd} - 1. \quad (06\text{-}1)$$

Then any $N_{Nd} - 1$ nodes in the scheme will be independent, and the last node will be dependent.

Let's now consider the construction of a system of independent meshes using the example of the same skeleton scheme.

06-6. An example of building a system of independent meshes

Building a system of independent meshes is somewhat more complicated. Let's consider it as an example of the same scheme. As already mentioned, a mesh is called independent if it includes at least one new branch that is not part of other meshes. Therefore, we will now begin to select meshes one by one and see if the next mesh includes at least one branch that is not included in the already considered meshes. The meshes can be chosen arbitrarily, but it is easier to do it on the "windows" formed by the branches as shown in Fig. 06-9.

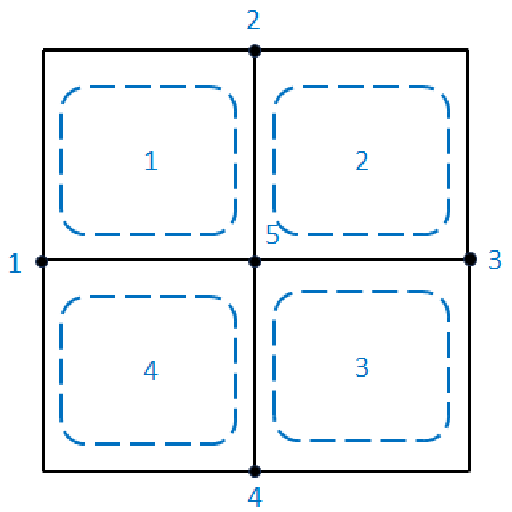


Figure 06-9. Electric circuit meshes

The results are entered in Table 06-2. As in the previous case, if the next branch under consideration has not yet been included in any of the considered meshes`, then we will highlight it in the table in green. If it is already found in the table earlier, we will mark it in red.

Table 06-2. System of independent meshes

Meshes	Branches	Independent?
1	1-2; 2-5; 1-5	Yes
2	2-3; 3-5; 1-5	Yes
3	3-5; 3-4; 4-5	Yes
4	1-5; 4-5; 1-4	Yes
5	1-2; 2-3; 3-4; 1-4	No

Consider mesh 1, which is formed by branches 1-2, 2-5 and 1-5. None of these branches is part of the previous meshes, because we consider this mesh first. Thus, it can be considered independent.

Mesh 2 is formed by branches 2-3, 3-5 and 1-5. The first two are not part of the previous mesh, so they are shown in green in the table. Branch 1-5 were included in the previous mesh, we show it in red. Mesh 2 contains two new branches, so it is an independent mesh.

Mesh 3 is formed by branches 3-5, 3-4 and 4-5. The last two branches have not yet been included in the two previous meshes, that is, mesh 3 is also independent.

Mesh 4 contains two branches (1-5 and 4-5) that were part of the previous meshes, and branch 1-4 is new. Therefore, mesh 4 is also independent.

But any other meshes will no longer be independent, because we have gone through each of the branches of the circuit at least once, and it is no longer possible to find a new branch. Thus, our scheme contains four independent meshes.

In graph theory, it is proved that the number of independent meshes N_{IM} (Independent Mesh) is related to the number of independent branches N_{IBr} (Independent Branch) and the number of independent nodes N_{INd} by the formula:

$$N_{IM} = N_{IBr} - N_{INd}. \qquad (06\text{-}2)$$

However, only the number of independent meshes can be calculated using this formula. To build a system of independent meshes, you need to use the definition of an independent mesh and the methodology outlined here.

06-7. Conclusions

1. Topology is a science that studies the laws of connection and mutual location of points, lines, surfaces, bodies and their parts in space, regardless of their sizes and properties
2. The main topological concepts of electric circuits are node, branch and mesh. A node is a junction of three or more branches. A branch is a part of a circle that is located between two nodes. A mesh is a closed path along independent branches of a circuit
3. The total number of different meshes in the scheme can be quite large
4. The concepts of independent branches, independent nodes and independent meshes are used in the theory of electric circuits
5. A branch that does not contain a current source is called independent
6. A node is said to be independent if it includes at least one new branch that is not included in all other nodes
7. A mesh is said to be independent if it includes at least one new branch that is not included in all other meshes.
8. The number of independent nodes is always one less than the total number of nodes in the scheme.
9. The number of independent meshes is related to the number of independent branches and independent nodes.
10. The system of independent meshes in a circle should be chosen according to the definition of an independent circuit.

Now we know all about the basic concepts of electric circuits. The time has come to study the basic laws to which all electrical circuits are subject without exception. This will be the topic of our next lecture.

LECTURE 010-07

BASIC LAWS OF ELECTRICAL CIRCUITS. KIRCHHOFF'S FIRST LAW

Lecture plan:

- Conclusions of the previous lecture
- Basic laws of electric circuits
- Kirchhoff's first law. General and special cases
- Writing Kirchhoff's first law due to voltages on the circuit elements
- Conclusions

! **You can watch this lecture on the author's YouTube channel "Theoretical Basics of Electrical Engineering" at the link https://youtu.be/gIO8KkAeMrk**

07-1. Conclusions of the previous lecture

In the previous lecture, we learned about the main topological concepts used in the analysis of electric circuits. Such concepts are a branch, a node, and a circuit. A branch is a part of a circuit between two nodes. A node is a junction of three or more branches. And finally, a circuit is a closed path along the branches of the scheme.

Most often, when analyzing electric circuits, we will deal with independent branches, nodes and circuits. We will call a branch that does not contain a current source an independent one. Then its current can be any, and it is determined by the parameters of the elements of this branch and other branches. But when the branch contains a current source, its current is determined by this source and cannot take any other value.

An independent node is a node that includes at least one new branch that is not part of other nodes. We found out that the number of independent nodes in the scheme is always one less than the total number of nodes in it.

A circuit is called independent if it includes at least one new branch that is not part of other circuits. A contour can be formed only by independent branches. The number of independent circuits in the scheme can be calculated if the number of independent branches and the number of independent nodes are known.

The concepts of nodes and branches are used in the formulation of the basic laws of electric circuits, the study of which we begin. We will first give a general description of these laws, and then consider them in detail.

05-2. Basic laws of electric circuits

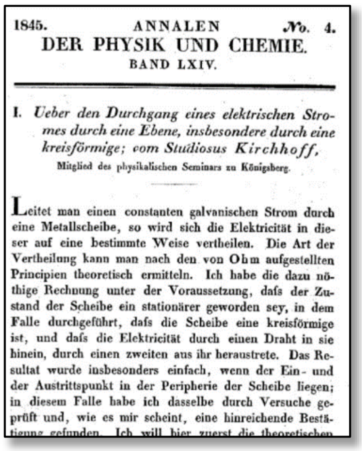
Surprisingly, the theory of electric circuits is based on only three basic laws. Remember, in lecture 010-02 we talked about the magic of the number "three", which we encounter in electrical engineering in completely unexpected places. We first encountered this mystical trio when we were studying the properties of the elements of electric circuits. Now this is the second meeting, but not the last.

These three laws that govern processes in all electrical circuits are Kirchhoff's first and second laws, as well as the commutation law.

The last law - the law of commutation manifests itself when an electric circuit operates in the mode of a transient process. For now, we will not deal with it, so we will focus on the study of the first two laws - Kirchhoff's laws.

Густав Роберт Кірхгоф
12.03.1824 – 17.10.1887

In 1885, in the magazine "Annalen der Physik" (Annals of Physics) German physicist Gustav Robert Kirchhoff published an article "Ueber day Durchgang eines electric Stromes through one Ebene, in particular through one kreisförmige" ("On the passage of an electric current through a plane, especially a circular one"). It was the first-time laws were formulated, which were later named after him - Kirchhoff's laws. To prove these laws, Kirchhoff had to use a rather complex mathematical apparatus - the theory of the electromagnetic field. But fortunately, we can use the final result - two laws that are formulated very simply.



1845. ANNALEN №. 4.
DER PHYSIK UND CHEMIE.
BAND LXIV.

I. *Ueber den Durchgang eines elektrischen Stromes durch eine Ebene, insbesondere durch eine kreisförmige; vom Studiosus Kirchhoff,*
Mitglied des physikalischen Seminars zu Königsberg.

Leitet man einen constanten galvanischen Strom durch eine Metallscheibe, so wird sich die Elektricität in dieser auf eine bestimmte Weise vertheilen. Die Art der Vertheilung kann man nach den von Ohm aufgestellten Principien theoretisch ermitteln. Ich habe die dazu nöthige Rechnung unter der Voraussetzung, daſs der Zustand der Scheibe ein stationärer geworden sey, in dem Falle durchgeführt, daſs die Scheibe eine kreisförmige ist, und daſs die Elektricität durch einen Draht in sie hinein, durch einen zweiten aus ihr heraustrete. Das Resultat wurde insbesonders einfach, wenn der Ein- und der Austrittspunkt in der Peripherie der Scheibe liegen; in diesem Falle habe ich dasselbe durch Versuche geprüft und, wie es mir scheint, eine hinreichende Bestätigung gefunden. Ich will hier zuerst die theoretischen

Kirchhoff's first law, which is also called the "law of currents", is formulated as the algebraic sum of currents acting in a node is equal to zero. Formulation of the second law - the algebraic sum of voltages in the circuit is zero. We will discuss Kirchhoff's second law in detail in the next lecture. Now let's study the first law in detail.

07-3. Kirchhoff's first law. General and special cases

Kirchhoff's first law establishes that the algebraic sum of currents acting in a node of an electric circuit is equal to zero. If we consider that electric current is the directed movement of electric charges, then it follows from Kirchhoff's first law that electric charges cannot accumulate in the nodes of a circle: the number of charges that arrive at a node is equal to the number of charges that left it.

In mathematical form, the equation of Kirchhoff's first law can be written as follows:

$$\sum_{k=1}^{N} i_k = 0. \qquad (07-1)$$

Please note that in this formula, the electric current is indicated by a small letter "i", that is, we are talking about the instantaneous value of the current. This means that Kirchhoff's first law holds for instantaneous values at any instant of time.

Usually, when we are talking about an algebraic sum, it means that its components can be included in the sum both with a "plus" sign and with a "minus" sign. Therefore, we need to formulate a rule according to which for each current we will determine its sign in the equation of Kirchhoff's first law. This rule can be, for example, the following: the current coming to the node will be taken with a "plus" sign, and the outgoing current - with a "minus" sign.

! **The current coming to the node in the equation of Kirchhoff's first law is taken with a "plus" sign, and the outgoing current - with a "minus" sign**

Let the currents act in some node of the circuit as shown in Fig. 07-1:

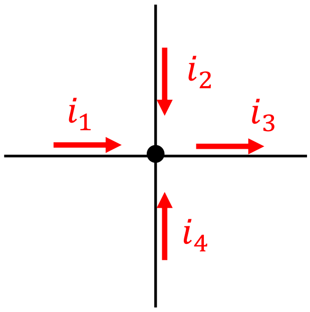


Figure 07-1. An example of compiling the equation of Kirchhoff's first law

Let's write the equation for this node according to Kirchhoff's first law (formula 07-2):

$$i_1 + i_2 - i_3 + i_4 = 0. \qquad (07\text{-}2)$$

As we can see from Fig. 07-1, currents i_1, i_2 and i_4 directed to the node, therefore, they are included in the equation (07-2) with a "plus" sign. The current i_3, on the contrary, leaves the node, so it is included in the equation with a minus sign. The algebraic sum of these four currents is zero.

The form of recording Kirchhoff's first law in the form (07-1) is a general case. It can be used for all electric circuits in all operating modes. But there is another form of writing Kirchhoff's first law, which is a special case of the previous one. Consider, for example, a node, which includes branches containing current sources (Figure 07-2). As we remember, such branches are called dependent branches.

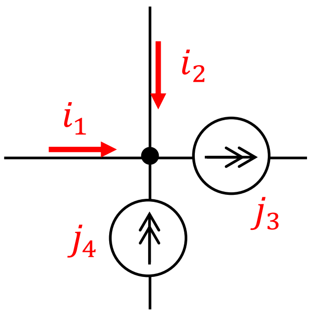


Figure 07-2. A node with dependent branches

Let's write the equation of Kirchhoff's first law for this node (formula 07-3):

$$i_1 + i_2 - j_3 + j_4 = 0. \qquad (07\text{-}2)$$

The current of the third branch j_3 known to us both in direction and magnitude. It comes out of the node, so it enters equation (07-3) with a minus sign. Current j_4 enters the node, we take it with a "plus" sign. However, these two currents are known and we can transfer them to the right-hand side of the formula:

$$i_1 + i_2 = j_3 - j_4.$$

In the general case, this equation can be written as:

$$\sum_{k=1}^{N} i_k = \sum_{k=1}^{N} j_k\,. \qquad (07-3)$$

According to the formula (07-3), Kirchhoff's first law can be formulated as "The algebraic sum of the currents of independent branches in a node is equal to the algebraic sum of the currents of the current sources entering this node." Please note that $j_3$we included the current that left the node in the right-hand side of the equation with a "plus" sign, and the current $j_4$that entered the node - with a "minus" sign. That is, the rule of signs for the right-hand side of equation (07-3) can be formulated as follows: if the current of the current source approaches the node, then it is included in the right-hand side of the equation of Kirchhoff's first law with a "minus" sign, and if it leaves the node, then with a sign "plus".

As you can see, this equation is a special case of equation (07-1) and can be used only when the node includes dependent branches.

07-4. Writing Kirchhoff's first law due to voltages on the circuit elements

Kirchhoff's first law establishes that the algebraic sum of currents in a node is equal to zero. Therefore, this law is often called the law of currents. But it can be written not only for currents but also for voltages on the elements of the branches that are connected to the node. Let's consider this possibility using the example of the circuit shown in Fig. 07-3.

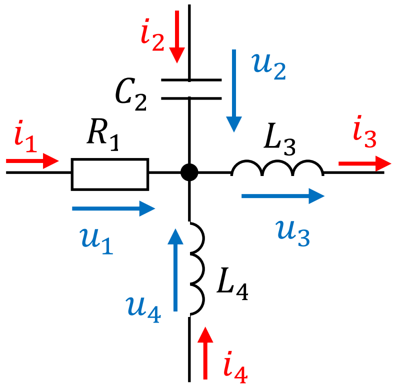


Figure 07-3. A node with connected elements

This scheme corresponds to the skeletal scheme that was shown in Fig. 07-1, but now the elements connected to the node are shown here. Previously, for this node, we formulated the equation of Kirchhoff's first law for the currents of the branches approaching the node:

$$i_1 + i_2 - i_3 + i_4 = 0.$$

But let's remember that from lecture 010-03 we know the relationship between voltages and currents of passive elements. These ratios were summarized in a separate table (see Table 03-1). These relations allow, in particular, to express the current of each passive element in terms of the voltage on it. So, we can write the equation of Kirchhoff's first law not through currents, but through voltages acting on the elements.

Consider the equation concerning currents. Current i_1, according to the diagram in fig. 07-3, acts in the resistor R_1. From Table 03-1, we remember that the current of a resistor is related to its voltage by the equation

$$i_R = G \cdot u_R = \frac{1}{R} \cdot u_R.$$

In our case, we can write:

$$i_1 = \frac{1}{R_1} \cdot u_1.$$

For the capacitorC_2 from table 03-1 we have:

$$i_2 = C_2 \cdot \frac{du_2}{dt},$$

for inductor L_3:

$$i_3 = \frac{1}{L_3} \int u_3 \cdot dt,$$

and for the inductor L_4:

$$i_4 = \frac{1}{L_4} \int u_4 \cdot dt.$$

Thus, the equation of Kirchhoff's first law for our node can be written as:

$$\frac{1}{R_1}\cdot u_1 + C_2\cdot\frac{du_2}{dt} - \frac{1}{L_3}\int u_3\cdot dt + \frac{1}{L_4}\int u_4\cdot dt = 0.$$

Note that this equation, unlike the equation for currents, is an integrodifferential equation.

07-5. Conclusions

1. All electric circuits obey only three basic laws - Kirchhoff's first and second laws and the law of commutation
2. All other interdependencies are consequences of these three basic laws
3. Kirchhoff's laws are universal, that is, they can be used for all types of electric circuits in all modes of operation
4. There are two formulations of Kirchhoff's first law. Formulation in the form $\sum i = 0$is basic and can be used in all cases
5. The formulation in the form $\sum i = \sum j$is a derivative of the main one and is usually used in equations for calculating electrical circuits that contain current sources
6. The equations according to Kirchhoff's first law can also be written in terms of stresses on the circle elements. In this case, they are integrodifferential equations
7. In the vast majority of cases, the number of unknown currents (or the number of independent branches in the circuit) is greater than the number of independent nodes. Therefore, only the equations according to Kirchhoff's first law are not enough to calculate these currents

In the next lecture, we will get to know Kirchhoff's second law in detail.

LECTURE 010-08

BASIC LAWS OF ELECTRIC CIRCUITS. KIRCHHOFF'S SECOND LAW

Lecture plan

- Results of the previous lecture
- Basic laws of electric circuits
- Kirchhoff's second law. General and special cases
- The equation of Kirchhoff's second law for the currents in the branches of the circuit
- Once again about duality in electric circuits
- Conclusions

! **You can watch this lecture on the author's YouTube channel "Theoretical basics of electrical engineering" at the link https://youtu.be/9BCRYo3a9yg**

08-1. Results of the previous lecture

In the previous lecture, we began to get acquainted with the basic laws of electric circuits. They are called basic because all electric circles in all modes of operation are obey them without exception.

There are only three such laws. These are Kirchhoff's first and second laws and the law of commutation. We will start using the first two laws from the next topic, when calculating electric circuits. The third law – the commutation law determines the operation of electrical circuits in the transient process mode. We will get to know him when we start studying the relevant topic.

From the last lecture, we know that Kirchhoff's first law has two formulations. The first of them – the algebraic sum of currents in a node of an electric circuit is zero, is the basic formulation that can be used always. But there is also a special formulation that establishes that the algebraic sum of the currents of independent branches in a node is equal to the algebraic sum of the currents of dependent branches. Remember, a branch that does not contain current sources is called independent.

We will sometimes use the formulation in the second form when calculating electrical circuits containing current sources.

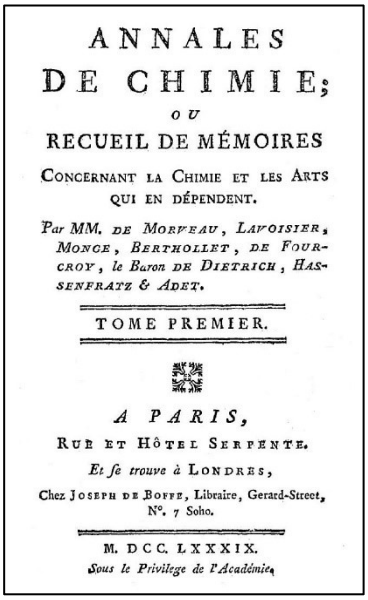
Kirchhoff's first law can be written not only for the currents in the branches of the circuit but also for the voltages on the elements of the branches that form the node. But at the same time, the equation of Kirchhoff's first law is integrodifferential.

Now we turn to the consideration of Kirchhoff's second law. But first, let's once again give a brief description of the basic laws of electric circuits.

08-2. Once again about the basic laws of electric circuits

As we already said in the previous lecture, the entire theory of electric circuits is built on only three basic laws – Kirchhoff's two laws and the commutation law. At the beginning of the study of electric circuits, we will need only two of Kirchhoff's laws.

For the first time, the German physicist Gustav Robert Kirchhoff announced the discovery of the two laws of electric circuits in 1885, when he published the article " Ueber day Durchgang eines electric Stromes through one Ebene, in particular through one kreisförmige " ("On the passage of an electric current through a plane, especially a circular one"). in the magazine " Annalen der Physik und Chemie " (Annals of Physics and chemistry). This scientific journal is one of the oldest in Europe, it has been published since 1789. Its founder and first editor was the famous French scientist Antoine Lavoisier. You have certainly heard this name. Initially, in 1789, the journal was called "Annals of Chemistry", then the name was expanded – "Annals of Physics and Chemistry", and later the journal was divided into two separate journals – "Annals of Physics" and "Annals of Chemistry". However the numbering of the issues did not change and is still carried out from the first issue. You can see a photocopy of the cover of the first issue of this magazine here. The Roman numerals MDCCLXXXIX on this cover correspond to the year 1789.


ANNALES
DE CHIMIE;
OU
RECUEIL DE MÉMOIRES
CONCERNANT LA CHIMIE ET LES ARTS QUI EN DÉPENDENT.
Par MM. DE MORVEAU, LAVOISIER, MONGE, BERTHOLLET, DE FOURCROY, le Baron DE DIETRICH, HASSENFRATZ & ADET.
TOME PREMIER.
A PARIS,
RUE ET HÔTEL SERPENTE.
Et se trouve à LONDRES,
Chez JOSEPH DE BOFFE, Libraire, Gerard-Street, N°. 7 Soho.
M. DCC. LXXXIX.
Sous le Privilège de l'Académie.

In this article, Gustav Robert Kirchhoff first formulated the laws that were later named after him. Note that Kirchhoff was only 21 years old at the time the article was published. To prove these laws, Kirchhoff had to use a rather complex mathematical apparatus – the theory of the electromagnetic field. But fortunately, we can use the result – two very simple laws.

08-3. Kirchhoff's second law. General and special cases

The formulation of Kirchhoff's second law in the general case is as follows: the algebraic sum of the voltages in the mesh is zero:

$$\sum_{k=1}^{N} u_k = 0. \qquad (08-1)$$

In the previous lecture, we already talked about the fact that in an algebraic sum each component can have either a "plus" sign or a "minus" sign. Therefore, before composing the equation according to Kirchhoff's second law, we must decide on the rule according to which we will put the signs in this equation.

In Kirchhoff's second law, we are talking about the meshes of an electrical circuit. And the mesh, as we remember, is a closed path along independent branches of the scheme. We can follow this path only in one of two directions – either clockwise or counter – clockwise. Let's agree that we will always go around the mesh clockwise. We will refer to this direction as the "conditional positive direction of bypassing the mesh".

Having chosen the conditional positive direction of the bypass, we can formulate the rule of signs in the equation of Kirchhoff's second law: if the direction of the voltage entering the circuit coincides with the conditional positive direction of the bypass, then in the equation we will put a "plus" sign in front of it. Otherwise, we will put a "minus" sign.

! **If the direction of the voltage coincides with the conditional positive direction of the circuit bypass, it is included in the equation of Kirchhoff's second law with a "plus" sign. If the directions do not match - with a "minus" sign**

Consider, for example, the mesh shown in Fig. 08-1.

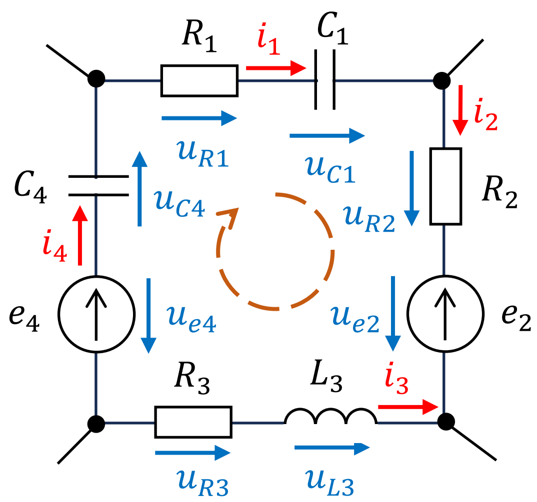


Figure 08-1. An example of an electrical mesh

Let us know the directions and magnitudes of the currents acting in the branches of this mesh, as shown in Fig. 08-1 with red arrows. In this case, we automatically become aware of the voltage directions on all passive elements of the mesh (blue arrows). Remember, in passive elements, the direction of current and voltage always coincide.

We also know the directions of the voltages on the active elements of the mesh – on the voltage sources! They are always oppositely directed relative to the directions of electromotive forces (EMF) of these sources.

By choosing the conditional positive direction of the mesh (shown by the dashed arrow inside the mesh), we can formulate the equation of Kirchhoff's second law for it. Let's start the traversal from the top left node. We have:

$$u_{R1} + u_{c1} + u_{R2} + e_2 - u_{L3} - u_{R3} - e_4 + u_{C4} = 0. \tag{08-2}$$

Let's explain the signs in this equation. Voltages directions on the elements of the first branch (u_{R1} and u_{C1}) coincide with the conditional positive direction of the mesh. That is why there are "plus" signs in front of them in the equation. The same can be said about the directions of voltage u_{R2} and u_{e2}. Note that the direction of the source EMF e_2 is opposite to the direction of the bypass, but the voltage on it coincides in the direction with the bypass and is equal to e_2.

Voltages directions on the elements of the third branch (u_{L3} and u_{R3}) are the opposite of bypass, they are preceded by "minus" signs in equation (08-2).

The direction of the voltage at the source e_4 is opposite to the bypass. In this equation, EMF e_2 presents with a "minus" sign. And, finally, the voltage u_{C4} coincides with the bypass, we put a "plus" sign in front of it.

When composing the equation according to Kirchhoff's second law, we can include in it not only individual voltages on individual elements of the mesh but also voltages acting on a set of elements, for example, on the branches of the circuit. Consider a branch with elements R_2 and e_2 from Fig. 08-1. Let us denote the nodes between which this branch stands as 1 and 2, and the voltage between them as u_{12} (Figure 08-2).

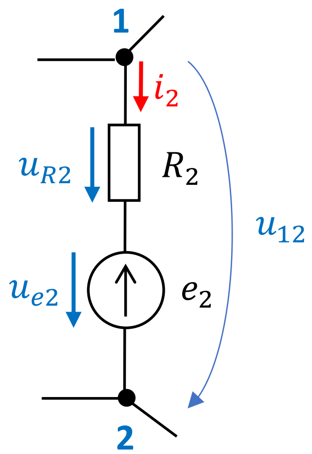


Figure 08-2. A separate branch of the scheme

Then we can compose the following equation of Kirchhoff's second law:

$$u_{12} - e_2 - u_{R2} = 0. \quad (08\text{-}3)$$

Here we started going around the mesh from node 1 in a clockwise direction. The direction of voltage on the branch u_{12} coincides with the direction of bypass, so we include it in the equation with a "plus" sign. In this way, we went from node 1 to node 2 in terms of voltage u_{12}. We close the mesh, rising from node 2 to node 1. At the same time, the directions of the voltages on the resistor R_2 and sources e_2 we meet as opposites of bypass. That is why in the equation there is a "minus" sign in front of them.

From equation (08-3), knowing any two voltages, one can easily obtain an unknown third. We will use Kirchhoff's second law in this form when we begin to calculate electrical circuits using the nodal voltage method.

The equation written as (08-1) is the basic form of Kirchhoff's second law. It can always be used for any electrical circuit.

But from this basic form, you can get a slightly different, special form of writing the equation of Kirchhoff's second law.

Consider equation (08-2) again. Note that the EMF of the voltage sources, e_2 and e_4 are known, because these are the parameters of the voltage sources. Therefore, we can transfer them to the right side of the equation:

$$u_{R1} + u_{c1} + u_{R2} - u_{L3} - u_{R3} + u_{C4} = -e_2 + e_4. \qquad (08\text{-}4)$$

In the left side of this equation, the voltages on the passive elements included in the mesh are recorded. In electrical engineering, the voltage that occurs on passive elements under the action of current through these elements is called as “voltage drop”.

! **The voltage on the passive elements of the electric circuit is called the “voltage drop”**

Using the term "voltage drop", we can write a special case of Kirchhoff's second law equation as:

$$\sum_{k=1}^{N} u_k = \sum_{k=1}^{N} e_k. \qquad (08-5)$$

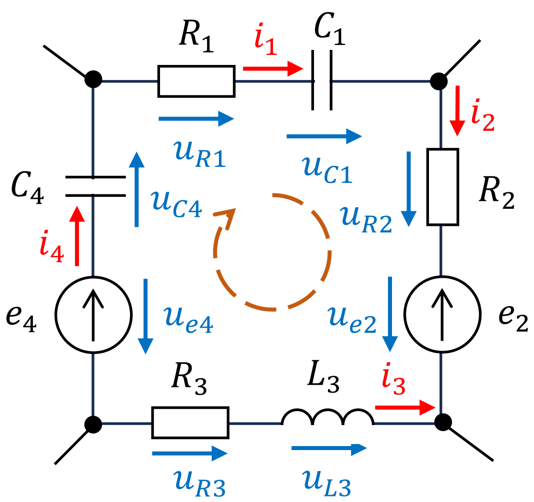
According to the formula (08-5), the algebraic sum of the voltage drops in the mesh is equal to the algebraic sum of the EMFs entering this mesh. This is a special form of writing the second Kirchhoff equation, which can only be used for a mesh that contains only passive elements and voltage sources.

It is from this formulation of Kirchhoff's second law that students usually get to know electrical engineering in high school or physics lectures. But you must remember that this is only a special formulation, which is a special case of the general formulation of Kirchhoff's second law.

When forming this equation, we transferred the EMF of the voltage sources to the right side of the equation with reversed signs. However, we remember that the voltage and EMF of the voltage source are oppositely directed. This fact allows us to formulate a separate sign rule for the right-hand side of the equation (08-5): the EMF of the voltage source in the right-hand side of the equation is taken with a "plus" sign if its direction coincides with the conditional positive direction of the mesh bypass. In the opposite case, a "minus" sign is placed before the EMF.

08-4. The equation of Kirchhoff's second law for the currents in the branches of the mesh

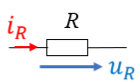
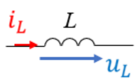
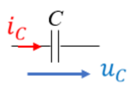
Kirchhoff's second law is formulated with respect to voltages acting on circuit elements. That is why it is also called the law of voltages. But further on, we will most likely need another form of writing Kirchhoff's second law, in which the voltage on each element of the circuit is expressed in terms of the current of this element. Let's consider this possibility on the example of the mesh shown in fig. 08-1. Let's repeat it here for convenience.



For this mesh, we composed the equation of Kirchhoff's second law in the general form (formula 08-2):

$$u_{R1} + u_{c1} + u_{R2} + e_2 - u_{L3} - u_{R3} - e_4 + u_{C4} = 0.$$

From lecture 010-03 we know the relationship between voltages and currents of passive elements. These ratios were summarized in a Table 03-1. For convenience, we will give this table again.

Елемент	$u = f(i)$	$i = f(u)$
Резистор 	$u_R = R \cdot i_R$	$i_R = G \cdot u_R$
Котушка 	$u_L = L \cdot \frac{di_L}{dt}$	$i_L = \frac{1}{L} \cdot \int u_L dt$
Конденсатор 	$u_C = \frac{1}{C} \cdot \int i_C dt$	$i_C = C \cdot \frac{du_C}{dt}$

The equations in the first column of this table allow us to express the voltage across each passive circuit element in terms of its current. In this way, we can rewrite the equation of Kirchhoff's second law in terms of the currents in the mesh branches.

So, for example, the voltage across the resistor R_1 can be written as:

$$u_{R1} = R_1 \cdot i_1,$$

voltage on the capacitor C_1:

$$u_{C1} = \frac{1}{C_1}\int i_1 dt,$$

and the voltage on the coil L_3:

$$u_{L3} = L_3 \frac{di_3}{dt}.$$

Having recorded the voltages on all mesh elements in a similar way, on the basis of (08-2) we obtain:

$$R_1 i_1 + \frac{1}{C_1}\int i_1 dt + R_2 i_2 + e_2 - L_3 \frac{di_3}{dt} - R_3 i_3 - e_4 + \frac{1}{C_4}\int i_4 dt = 0.$$

Thus, the equations of Kirchhoff's second law can be written in terms of currents acting in the branches of an electrical circuit. Very soon, this will allow us to perform calculations of electric circuits using Kirchhoff equations. But at the same time, we will have to face a great difficulty – because these equations are, in general, integrodifferential equations.

08-5. Once again about duality in electric circuits

From lecture 010-05, we learned that electric circuits are characterized by the phenomenon of duality. We know that using the phenomenon of duality, we can take a mathematical expression or definition that describes some real physical process, replace all quantities and concepts in them with corresponding dual quantities or concepts, and get as a result new expressions or definitions that correspond to another real physical process.

We composed a table of dual quantities and concepts (table 05-2 of lecture 010-05). We also said that the given table is not complete. Now we have an opportunity to supplement this table.

Let's repeat the definitions of independent nodes and independent meshes that were given in lecture 010-06:

- a node that includes at least one new branch that is not part of other nodes is called independent;
- a mesh that includes at least one new branch that is not part of other meshes is called independent.

If in the definition of an independent node we replace the concept of "node" with the concept of "mesh", then we will get the correct definition of an independent circuit as a result. This shows that node and mesh are dual concepts.

Let's look at the formulation of Kirchhoff's first and second laws, which we learned about in this lecture:

– the algebraic sum of the currents in the node is zero;
– the algebraic sum of the voltages in the mesh is zero.

We already know that current and voltage, as well as node and mesh, are dual concepts. By replacing "current" with "voltage" and "node" with "mesh" in the definition of Kirchhoff's first law, we get the equation of Kirchhoff's second law. Thus, Kirchhoff's first and second laws are dual.

Let's display the new dual concepts and laws in the table of dual concepts (table 08-1).

Table 08-2. Dual concepts and quantities in electrical engineering

Concept (quantity)	Dual concept (quantity)
High-voltage	Current
Resistance	Conductance
Inductance	Capacity
Inductor	Capacitor
Voltage source	Current source
EMF of the voltage source ($\boldsymbol{E}$)	Current source current (J)
Mesh	Node
Kirchhoff's second law	Kirchhoff's first law

Note that this table is not yet complete, and will be supplemented when studying the theoretical foundations of electrical engineering.

08-6. Conclusions

All electric circuits obey only three basic laws – Kirchhoff's first and second laws and the law of commutation

All other interdependencies in electrical circuits are consequences of these three basic laws

Kirchhoff's laws are universal, that is, they can be used for all types of electric circuits in all modes of operation

There are two formulations of Kirchhoff's second law. Formulation in the form $\sum u = 0$ is basic and can always be used

The voltage that occurs on passive elements of circles due to the action of electric current past them is called "voltage drop"

The formulation of Kirchhoff's second law in the form $\sum u = \sum e$ is derived from the first and is usually used in equations for calculating electric circuits

The mesh and node of an electrical circuit are dual concepts, and Kirchhoff's first and second laws are dual

In general, the system of equations according to Kirchhoff's laws is a system of integrodifferential equations.

With this lecture, we completed the consideration of the topic "Basic concepts and laws of electric circuits". The next lecture will open a new topic - "Electric circuits of direct current".