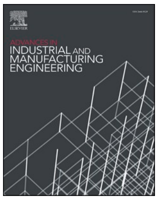
Advances in Industrial and Manufacturing Engineering 8 (2024) 100135

Contents lists available at ScienceDirect

Advances in Industrial and Manufacturing Engineering

journal homepage: www.sciencedirect.com/journal/advances-in-industrial-and-manufacturing-engineering



A gear grinding allowance stochastic simulation model

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ARTICLE INFO

Keywords:
Gear grinding allowance
Ergodic random process
Stationary random process
Stochastic random process
Ensemble of realizations

ABSTRACT

The paper proposes a method and technique that allow, using a minimum sample (3 units) from a batch (30 units) of real gears, to obtain stochastic information about the actual distribution of the gear grinding allowance both on the left and right flanks of the gear gaps for all gears in the batch. The proposed gear grinding allowance stochastic model is based on the representation of the distribution of the gear grinding allowance along the gear periphery as a superposition of a sinusoidal component with a random amplitude and a random component of the white noise type. The conditions for the equivalence of the stochastic characteristics of real (measured) and virtual (not actually measured) gears are formulated, namely the ratio between the amplitude of the sinusoidal component and the average amplitude of the high-frequency harmonic components of the gear grinding allowance distribution along the gear periphery. The gear grinding allowance stochastic model adjusted according to the proposed method makes it possible to predict the distribution of the gear grinding allowance on the left and right flanks of the gear gaps for a batch of gears and, taking into account the information obtained (replacing the experimental data), both evaluate the existing technology and optimize the gear grinding allowance value. As a result, it becomes possible to assess the quality of the technological process of manufacturing the gear, taking into account the effect random deformations to gear grinding allowance at the stage of their heat treatment (hardening), which is performed before the gear grinding operation. In turn, optimization of the grinding allowance (based on its actual distribution along the periphery of the gear) makes it possible to reduce both defects in burns (with increased allowance) and defects in unground teeth (with insufficient allowance).

1. Introduction

The technological process of manufacturing a gear includes dozens of technological operations, including turning, gear hobbing, internal grinding and gear grinding. The most labor-intensive machining operation is gear grinding, which takes from 40% to 70% of the total machine time. Therefore, the efficiency of the gear grinding operation determines the efficiency of the entire technological process for manufacturing a batch of gears. It is known that the efficiency of the gear grinding operation largely depends on the reasonable assignment of allowances on the side surfaces of the teeth.

It has been established experimentally that the existing practice of assigning allowances for gear grinding operations using reference tables is inaccurate, since it does not take into account the conditions of the technological process in the previous and ongoing operations. To assess the labor intensity and cost of gear grinding, as well as to make a decision on process optimization, it is necessary to know the amount of gear grinding allowance (GGA) for all gears included in the batch. This procedure is labor-intensive, especially for large batches. When developing a technological process for manufacturing a gear, there is a standard (nominal) allowance provided by the technologist.

Allowances for the gear grinding operation of cylindrical gears are assigned, as a rule, according to the reference tables and are set in the direction perpendicular to the side surface of the tooth (Kalashnikov, 2006). These data are based on the experience of mechanical engineering production and do not reflect the current state of the technological process in a particular production. In addition, the allowance for gear grinding, in contrast to the allowance for flat and cylindrical grinding, is an ambiguous characteristic of the thickness of the cut layer due to the complex shape of the tooth gap profile. This leads to the dependence of the size of the allowance on its location on the tooth profile and on the direction of its measurement relative to the tooth surface (perpendicular, vertical or horizontal in the coordinate system of the tooth or tooth gap).

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https://doi.org/10.1016/j.aime.2023.100135
Received 25 February 2023; Received in revised form 23 October 2023; Accepted 29 November 2023

The GGA is affected by the distortion of the shape of the gear blank due to the effect of chemical-thermal treatment on it. This treatment is accompanied by unpredictable deformations of a workpiece with a complex-shaped profile (shrinkage, stretching, bending, torsion and their combinations), which lead to a change in the thickness of the cut layer during the subsequent operation of grinding the hardened surfaces of the teeth.

The GGA value should be minimal, but at the same time sufficient to compensate for the influence of factors that affect the amount of the GGA. One of these factors is the residual deformation of the gear blanks after their hardening, which is performed before the gear grinding operation. To optimize the GGA in the factory, labor-intensive experimental statistical studies of the actual distribution of the GGA along the periphery of the gear for all gears of the batch are required.

A gear after its heat treatment (hardening) is deformed. As a consequence, before the gear grinding rough stage the GGA on the teeth flanks is unequal. Statistical studies have shown that over the gear periphery the GGA changes along a sinusoid. As a result, the grinding wheel "grinds air" on some working strokes as well as "grinds hardly" when the GGA is great.

The GGA, in contrast to the grinding allowance for flat and cylindrical grinding, is an ambiguous characteristic of the depth of cut due to both the complex shape of the tooth profile and its shift along the periphery of the gear. This leads to the dependence of the GGA on its location on the gear profile, on the direction of its measurement relative to the tooth surface (perpendicular, vertical or horizontal in the coordinate system of the tooth gap) and on the location of the gap on the gear periphery. According to the gear drawing, the GGA remaining for tooth grinding is placed equidistantly in relation to the finished profile, i.e., it is kept constant in the direction normal to the profile of each tooth. In general, an equidistant gear grinding allowance on the complex-shaped part theoretically allows during the subsequent heat treatment providing a uniform layer of reinforced (hardened) material at any point of the part complex profile.

With profile gear grinding, the amount on individual sections of the toothed surface is unambiguously undefined. Firstly, it is caused by the influence of the error of the setup of the gear on the faceplate of the grinder in which the center of the gear does not coincide with the center of faceplate rotation. This error leads to the sinusoidal curve of the GGA change on the left and right flanks of the gear gaps. The period of this sinusoid is deterministic as it is determined by the angle 2π, since a gear is a body of rotation. The amplitude of this sinusoid is random as it is caused by the kinematic eccentricity of the previous operation and the geometric eccentricity of the current one, which are added vectorially. Secondly, the GGA is affected by the distortion of the gear due to the effect of chemical-and-thermal treatment on it, for example, the effect of carburizing and hardening. Such a treatment is accompanied by unforeseen deformations of the complex-shaped workpiece (shrinkage, stretching, bending, twisting and their combinations), which leads to a change in the thickness of the GGA to be removed in the next gear grinding operation. That is why the GGA amount on the left and right side of the gear teeth includes both deterministic and random components. Therefore, the paper uses a probabilistic approach to develop a GGA stochastic model.

Therefore, the purpose of the article is to develop a stochastic gear grinding allowance model, which is trained (or adjusted) based on the results of selective measurement of the GGA on the lateral sides of the teeth along the periphery of several (for example, three pieces) gears, selectively taken from a batch (e.g., a batch of thirty pieces).

Such a model will make it possible to generate a so-called ensemble of realizations of a random process. Each realization is the GGA distribution by gear angle of rotation from 0 to 2π. In this case, GGA signals for unmeasured gears must be generated according to a stochastic model, taking into account information about the stock of measured gears. It becomes possible to take into account the influence of random disturbances on the amount of allowance without measuring the allowance.

2. Literature review

A number of works aimed at studying the gear distortion problem and finding remedial measures (George et al., 2014; Khade and Ramgir, 2015; Gadagkar et al., 2013; Herring et al., 2004; Rakhit, 2000). The problems of runout, profile mill and lead and profile were analyzed during the study of distortion of the cooler fan drive gears of the ALH helicopter to solve a critical problem of rejection due to stresses acting to the gears (George et al., 2014). It has been established that runout is the main problem that occurs for cooler fan drive gears after improper (wrong) quenching (hardening). Therefore, efforts are made to prevent runout during quenching. Two types of fixtures have been developed for this purpose. One of the fixtures is needed at the time of carburizing operation to put a dead (motionless) weight on the component to avert the movement from the vertical position. Another fixture is needed at the time of hardening followed by quenching to prevent direct contact of the quenchant to affect the component.

Investigating the causes of distortion, mainly due to quenching that includes cooling rates, quenching mediums, and fixtures had been done (Khade and Ramgir, 2015). Three causes of deformation during carburizing heat treatment are established:

1) correct quenching oil temperature and flow rate selection,
2) modification of fixture design or arrangement of the components in the fixture,
3) modification design of baffles to control flow velocity.

The realization of certain technological and design solutions within the framework of the study of each cause contributed to the reduction of deformation.

The many parameters affect the distortion but the most predominant is the heavy weight of the gear (Gadagkar et al., 2013). In the process of analyzing the effect of gear weight on distortion problem it was identified the following three dominant parameters to reduce the distortion:

1) use of an improved fixture to hold the gear during heat treatment,
2) better cooling mechanism,
3) reduction of gear weight by making gear design modification.

The importance of normalizing, part size, equipment design, quench variables and quench tank design, material and their influence on gear wheel deformation is also considered (Herring et al., 2004).

There are two types of distortions that occur in gears (Rakhit, 2000). One is a body distortion, which, for gears, is measured in terms of out-of-roundness, out-of-flatness, or run-out dimensions. The second is the gear tooth geometry distortion. The gear distortion analysis was made after the heat treatment process in terms of tooth thickness, face width, addendum and dedendum circle diameters (Guterres et al., 2017). Distortion change ranges are established for the parameters specified above. It was found that the heat treatment gear distortion is very high and that after heat treatment, the dimensions of the addendum and dedendum circles diameters on the periphery of the gear change significantly.

When determining the grinding allowance for even simpler surfaces than gear ones, for example, flat and cylindrical ones, the distortion factor of the gear to be ground is usually taken into account by a corresponding increase in the GGA for the grinding operation resulting from chemical-thermal treatment. But due to hardening distortions, the GGA amount is not constant both over the tooth flanks and over the height of the tooth in one gap, and over the gear, i.e., within the gear revolution resulting in a considerable amount of stroke length not grinding the gear (Turich et al., 2009).

The GGA change law can be represented by some function of the coordinates of the points of the gear section to be ground. For example, during profile gear grinding, the GGA on the side of the processed profile changes according to a law close to a sinusoidal one (Turich et al., 2009; Clufl, 1997; Undewiss and Miller, 2010).

Gear grinding of carburized and hardened gears is performed due to correct positioning of a grinding wheel in the tooth gap. It is necessary to take into account the specified GGA unevenness along the height of the tooth and along the periphery of a gear (Lishchenko et al., 2023). For this purpose, modern grinding machines in production have built-in systems for measuring the GGA with a probe or grinding wheel on the number of right and left sides of the teeth (specified by the operator) in the position (specified by the operator) along the height of the tooth-wheel rim (Rakhit, 2000; Miller, 2017).

After measuring the GGA on the left and right flanks of the teeth around the periphery of the wheel, the intelligent software finds the best position of the grinding wheel between the teeth (for example, so as to make the minimum GGA on the left and right sides of the teeth equal).

The integrated measuring system feature for aligning the gear teeth allows measuring before grinding several selectable teeth around the gear along the lead to determine the GGA mean, GGA amount, and the minimum and the maximum GGA amounts (Clufl, 1997; Larshin et al., 2022). The software also identifies flanks that may not be completely ground. This allows the operator to decide whether grinding is advisable (Undewiss and Miller, 2010). Also, knowing the allowance on the sides of each gap, it's possible to avoid "grinding air" by determining the moment of contact between the wheel and the workpiece with an acoustic emission sensor.

In the technological process of manufacturing gears, unpredictable disturbances may arise that affect the amount of GGA for the gear grinding operation. Such disturbances include instability of heat treatment modes, type of heat treatment, physical and mechanical properties of the gear material, etc. To assess the effectiveness of the technological operation of gear grinding, taking into account the influence of the entire complex of disturbances on the GGA, it would be necessary to carry out measurements of all gears in the batch, which requires considerable time.

It is known that one of the methods for assessing the quality of a product is selective control, i.e., a method of decision-making for assessing the quality of all products by selectively measuring individual copies of the product. Obviously, this method can be extended to the assessment of GGA, including the use of a stochastic model for GGA (Lishchenko, 2018) and the other allowances (Vasin, 2006). However, there is no information on this approach in the literature. There is also no information about the statistical approach to determining GGA, according to which the GGA value is a superposition of the sinusoidal component of GGA (with a deterministic sinusoid period) and a random process such as "white noise".

3. Research methodology

It follows from the literature (George et al., 2014; Khade and Ramgir, 2015; Gadagkar et al., 2013; Herring et al., 2004; Guterres et al., 2017) that, despite the identification of the causes of and the measures taken to reduce the distortion, gears still have unpredictable deformation depending on a large number of factors: material, heat treatment conditions, gear design, etc.) These factors affect the distortion amount, which can be determined only by measuring the actual GGA before grinding, so as not to assign an inflated allowance for the operation, which leads to a decrease in the labor intensity of the operations.

It is known that a random process is a stochastic one if it has the properties of ergodicity and stationarity [stochastic random process]. Therefore, in further studies the hypothesis of a stochastic distribution of the GGA on the left and right sides of the gear gaps was adopted and that this distribution corresponds to a stochastic model of a random process.

Then it is necessary to justify the fact that the random component (GGA distribution) is the realization of an ergodic stationary random process in accordance with the properties of the random process on real gears. Such properties include the following (Gardner, 1990; Hopgood, 2017):

- the amplitude spectrum of a random process tends to disappear;
- the ergodicity of a random process, which consists in the fact that the average over many realizations with a 100 % probability coincides with the average over time of one realization of the process.

This allows for a limited length of one realization (for a gear, the length of one realization is equal to the central angle 2π) to replace its time average with the average over many realizations. In the theory of random processes, a set of realizations is called an ensemble of realizations, which defines a random process. In other words, if the length of the realization of a random process is insufficient (for the gear, the length of the spatial interval of the GGA change is 2π), it can be specified by a mathematical model - an ensemble of realizations, and the average over time can be replaced by the average over the set. This is caused by the fact that for stationary ergodic processes, the statistical characteristics obtained by averaging over many realizations and over time for one realization of a random process are equal to each other. Confirmation of this property is the well-known Parseval's or Rayleigh's theorem.

If proving that the real grinding allowance signal can be replaced by a virtual signal, which is a sine wave with white Gaussian noise superimposed on it, then it can be used in the design of a gear grinding technological system. It becomes possible to take into account the impact of the disturbance, which is an unforeseen fluctuation of the GGA, and then perform simulation modeling of the system. This makes it possible to develop a statistical model of GGA containing deterministic and random components.

The amplitude spectrum of a random process due to random phase ratios of frequency components has a tendency to disappear, which is caused by the compensating interaction of harmonics with random phases when the random process is sufficiently long. Further research is based on the specified property of the disappearance of the amplitude spectrum of a random process.

The output signal about the GGA can be recorded, for example, for the left flank of the measured gear, in the form of a discrete functional dependence

$$f(n) = z^{L}_{rated}(n), \tag{1}$$

where z^{L}_{rated} is normalized signal of left GGA, mm; n is the current number of the gap.

For the measured gears on the grinder HOFLER RAPID 1250, the corresponding time and frequency dependences for gear No. 1, gear No. 2, and gear No. 3 can be presented in the form

$$f_1(n) = z^{L1}_{rated}(n), F\{f_1(n)\} = F\left\{z^{L1}_{rated}(n)\right\} \tag{2}$$

$$f_2(n) = z^{L2}_{rated}(n), F\{f_2(n)\} = F\left\{z^{L2}_{rated}(n)\right\} \tag{3}$$

$$f_3(n) = z^{L3}_{rated}(n), F\{f_3(n)\} = F\left\{z^{L3}_{rated}(n)\right\} \tag{4}$$

The time average of the signals $f_1(n) = z^{L1}_{rated}(n)$, $f_2(n) = z^{L2}_{rated}(n)$, $f_3(n) = z^{L3}_{rated}(n)$, that is, from the signal of the following type

$$f_{ave}(n) = \frac{1}{3}\sum_{i=1}^{3} z^{Li}_{rated}(n) \tag{5}$$

This Fourier transform, that is, the spectrum of the average value of the GGA signal, can be represented as

$$F\{f_{ave}(n)\} = F\left\{\frac{1}{3}\sum_{i=1}^{3} z^{Li}_{rated}(n)\right\} \tag{6}$$

4. Results

An experimental study of the GGA (as an example, gear No. 1 in Fig. 1 is shown) made it possible to obtain the corresponding signals of the spatial (analogous to the term "temporal") GGA and the spectrogram of the left-flank GGA for three gears (Fig. 2, a) and the time average spectrogram (Fig. 2, b). Gear parameters: the number of teeth $z = 29$; module $m = 7$ mm; addendum circle diameter $da = 233.4$ mm; base circle diameter $db = 199.3473$ mm; root circle diameter $df = 207.0270$ mm; face width $B = 60$ mm; profile shift coefficient $x = 0.545$; pressure angle $\alpha = 20^\circ$; helix angle $\beta = -18^\circ$.

The last spectrogram (Fig. 2, b) was obtained in the form of a Fourier transform from the time average of the signals using equations (5) and (6). To implement the transformations (5) and (6), a virtual instrument was designed in LabVIEW 8.6 (Fig. 3).

To work in LabVIEW 8.6 with GGA signal, it is possible to use the Simulate Arbitrary Signal blocks, which are called from the Functions. When Simulate Arbitrary Signal block is clicked, Configure Simulate Arbitrary Signal window opens, in which it is possible to select Define Signal. In the window that opens, Define Signal, it is possible to assign a signal (X, Y) from the keyboard or by reading the created text file of the signal saved in ".lvm" format (readable and edited in Excel). In this case, all GGA signals were saved in appropriate files with ".lvm" format and loaded using Load Data in the Define Signal window. In block 2, the fast Fourier transform procedure is performed.

A graphic image of the output signal about the GGA in the form of its amplitude (Fig. 2) and phase spectrum is displayed on the front panel of the virtual instrument. This data output is performed using the Waveform Graph display blocks. These blocks are located in the Controls → Graph Indicators. Signals are propagated through the block diagram using virtual communication lines 6.

The output GGA signal $f_{ave}(n)$ (Fig. 4), generated by the virtual instrument (Fig. 3), is similar to the allowance signal for the gear No. 1 (Fig. 1, left flank). From Fig. 2, d follows that the spectrum of the averaged assumed random component has a weak tendency to disappear. However, it is necessary to have a sufficient number of realizations of the random process, three realizations are not enough.

An evaluation indicator is formed for each spectrogram:

$$\eta = \frac{A_1}{A_{ave}}, \tag{7}$$

where A_1 is the amplitude of the first harmonic with a period of 2π; A_{ave} is the average amplitude of harmonics, except for the first.

Moreover, $A_{ave} = \frac{1}{\left(\frac{z}{2}\right)-1} \sum_{i=2}^{z/2} A_1$, where z is the number of gear teeth.

The value of the upper summation limit z/2 is due to the fact that, according to the sampling Nyquist-Shannon sampling theorem, the number of discrete points in the frequency domain is two times less than the number of discrete points in the time domain. The following values of the evaluation indicators were obtained for the left η^L and right η^R flanks of the gaps (an analogue of the known signal-to-noise ratio).

Number of the gear	1	2	3	Ave
η^L	7.82	8.98	10	9.61
η^R	8.78	6.28	7.84	8.48

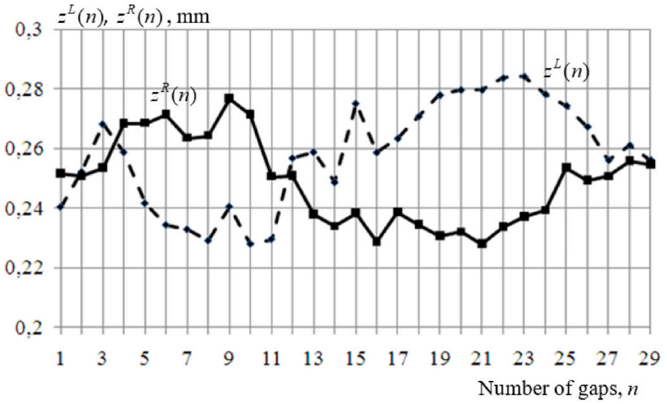

Fig. 1. Results of the GGA measurement on a CNC machine Höfler Rapid 1250 (z^L, z^R GGA readings on the left and right flanks).

To increase the number of realizations, i.e., to create an ensemble of realizations, a simulation GGA model containing systematic and random components was developed. In this model, each gear with the number of teeth (or gaps) z is considered as a realization of a stochastic process that contains systematic and random components (one of the possible ensemble of realizations). The systematic component of the GGA signal is realized by a harmonic sinusoidal function $y = \sin(x)$ (frequency $f = 10$ Hz) with a noise component superimposed on it. The noise component is white Gaussian noise with a standard deviation of σ_{noise}.

This simulation model allows to create virtual gears (separate realizations of the allowance signal) with different proportions of the random component in the stochastic two-component signal. This proportion is adjusted using the standard deviation σ_{noise}. The model will allow generating realizations of a random process - a random component of the allowance signal.

The simulation model was created using a virtual instrument in the LabVIEW 8.6 program (Fig. 5). The adjustment of the simulation model (Fig. 5) was performed in accordance with the received estimates of the allowance spectra of three real gears. The effect of the virtual model setting parameter on the value of σ_{noise}. For this, a 3-channel virtual device (Fig. 6) was developed, that is, a model of the ensemble of possible realizations of the GGA (one realization of the stock allowance is a one-flank allowance of one gear, the ensemble of realizations of the GGA corresponds to the batch of gears).

To present three realizations, which include a sinusoidal signal and white Gaussian noise in the LabVIEW 8.6 program, work with the Simulate Signal blocks, which are called by the Functions. In Fig. 6, blocks (icons) 1, 2, and 3 are models for the first, second, and third realizations of the two-component stock allowance signal. In the addition node (Add) 13 three realizations are added. The addition node is called through the Functions → Arithmetic and Comparison → Numeric. At the output of the addition node 13, the total realization is received, which enters the input of the Divide distribution node (node 14 in Fig. 6). At the output of node 14, the averaged realization is obtained (by dividing the total realization by three). The output signals of blocks 1, 2 and nodes 14 are sent to the corresponding Spectral Measurements blocks (blocks 4, 5), which perform a Fast Fourier Transform, converting the time representation of signals at a time interval of 0.1 s (29 readings at a frequency of 290 Hz) in the corresponding frequency representation. Blocks 4, 5 and 6 are called through the palette Functions → Signal Analysis → Spectral Measurements.

Moreover, at the output of block 6, the spectrum of the time-averaged realization is formed. Individual realizations in the time domain and the obtained results in the form of spectrograms are displayed on the front panel of the virtual device using the Waveform Graph display blocks (blocks 7, 8, 9, 10 in Fig. 6). The time-averaged signal of the allowance is displayed on block 11, and its spectrum on block 12. These blocks (11 and 12) are located in the Controls (Front Panel) → Graph Indicators palette.

By changing the level σ_{noise} in the range indicated below, the estimated parameters η_m at $m = 1$, 2 and 3 are obtained.

m	1	2	3
σ_{noise}	0.1	0.5	1
η_m	16.99	7.45	4.068

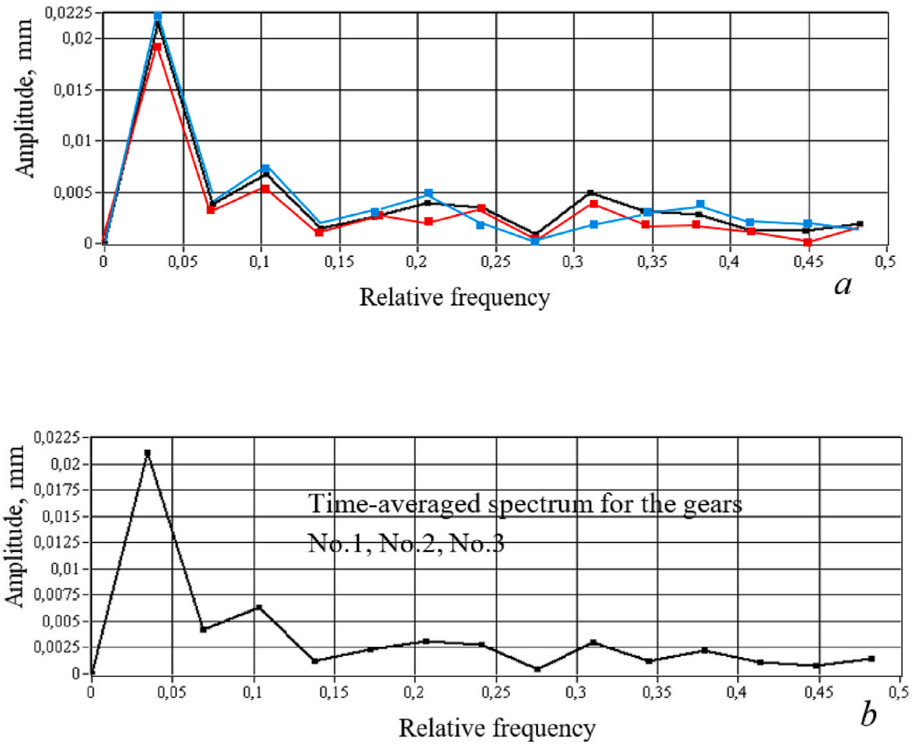

Fig. 2. Amplitude spectra of normalized left-flank GGA signals: a - black line – gear No. 1, red line – gear No. 2, blue line – gear No. 3, respectively; and b - for the averaged time signal of the GGA. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

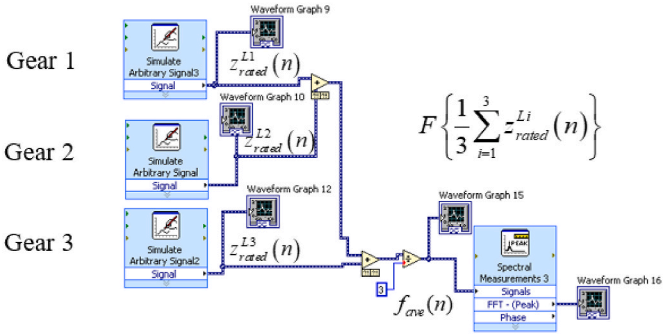

Fig. 3. Virtual instrument for averaging three signals and forming the spectrum of the average.

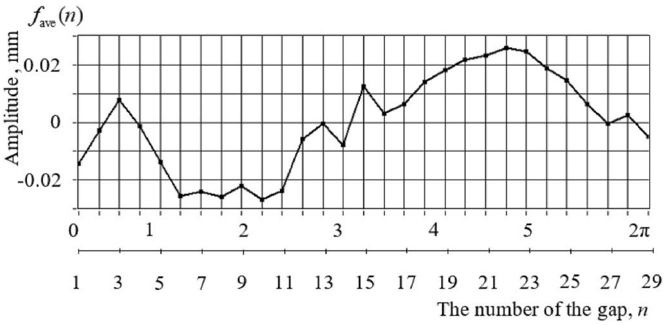

Fig. 4. Time-averaged left-flank GGA signal $f_{ave}(n)$.

The parameter η_m at $\sigma_{noise} = 0.5$ can take the value 7.45 … 8.67 in different realizations of the random process, which corresponds to the actually measured three gears.

The number of points in the realization of the simulation model of the stock allowance corresponds to the number of teeth of the real gear, i.e., $z = 29$. It can be taken for comparative evaluation, the amplitude of the sinusoidal periodic component $y = \sin(x)$ (frequency $f = 10$ Hz) for the realizations to be the same, for example, one. According to the

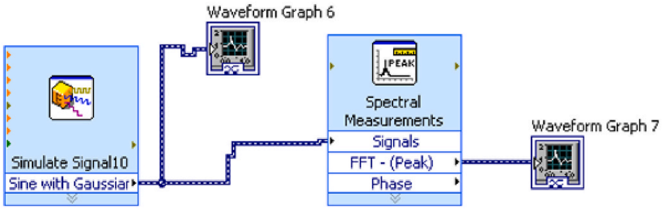

Fig. 5. Virtual instrument for simulation of a two-component signal of GGA.

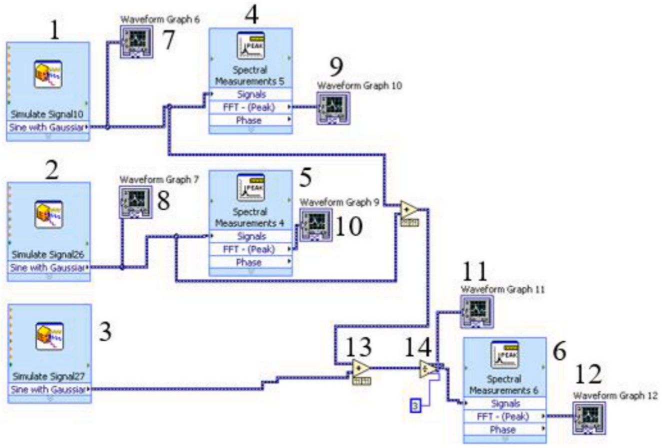

Fig. 6. 3-channel virtual instrument for simulation of a two-component signal of GGA.

selected level $\sigma_{noise} = 0.5$, e.g., three realizations of the additive simulation model of the gear (Fig. 7, a, b, c), and time-averaged realization (Fig. 7, d). their corresponding amplitude spectrograms (Fig. 8, a and b) and the averaged spectrum (Fig. 9).

It can be seen that the realization spectrum, which is the time average of the previous three realizations, has a tendency to decrease the amplitudes of the random component of the allowance signal (Fig. 8, b). However, the spectrum of the random component does not disappear with three realizations. For a more obvious demonstration of this tendency, it is necessary to increase the number of averaging channels, i.e., make this number more than three. This is due to the fact that a real random process can be implemented by increasing the number of realizations (formation of the average over the set), and the true spectrum appears in the limit. In practice, this means that it is necessary to increase the number of investigated gears and, accordingly, add more initial realizations of the measured values of the allowance.

To confirm the trend on the model, it is enough to increase the number of averaging channels for the virtual instrument in Fig. 6. By analogy with the 3-channel virtual instrument in Fig. 6, which implements the additive model of the GGA, a virtual instrument has been developed for a larger number of simulated gears, for example, a 25-channel (by analogy with Fig. 6, not shown). Now it is clear that the range of the time-averaged signal for 25 simulated gears tends to disappear (Fig. 9, red line).

This proves the random nature of the component in the normalized GGA signal. The systematic component is clearly expressed at a frequency of 10 Hz (Fig. 9) and does not undergo significant change both in individual realizations (Fig. 8) and for the averaged signal (Fig. 9).

5. Discussion

The ergodic property of a stationary random process is that each of its individual realizations (the distribution of GGA over the teeth on a separate gear according to the angle of rotation from 0 to 2π) is a characteristic representative of the entire set of possible realizations (the distribution of GGA for other gears in the same batch of gears), i.e., one realization (of sufficient duration) replaces many realizations during data processing. Moreover, for any fixed angular coordinate, the average GGA value over many realizations will be equal to the average GGA value measured over all teeth of the gears when it is rotated in the range from 0 to 2π. Thus, for an ergodic random process, averaging over many realizations (each realization is one gear) can be replaced by averaging over time within one realization (the average value of the GGA within one gear when it is rotated in the range from 0 to 2π).

Unpredictable changes in the GGA obviously cannot be described by the apparatus of deterministic functions. However, there are the following circumstances to determine the structure of the stochastic GGA mathematical model. Firstly, in serial production conditions, gears are sequentially machined in batches of about 30 pieces in each batch. This circumstance unites all the gears from one batch into one family, thereby not refuting the validity of the hypothesis about the ergodicity and stationarity of the random process. Secondly, the gears to be machined, in terms of their shape, belong to bodies of rotation. This circumstance confirms the hypothesis adopted about the presence in the allowance of a sinusoidal (deterministic along the length of the realization) component with a random amplitude, the period of change of which corresponds to one full revolution of the gear.

It is this circumstance that causes runout of the gear wheel, which in turn is caused by eccentricity when installing the gear on the mandrel during grinding (on a gear grinding machine) and when measuring the

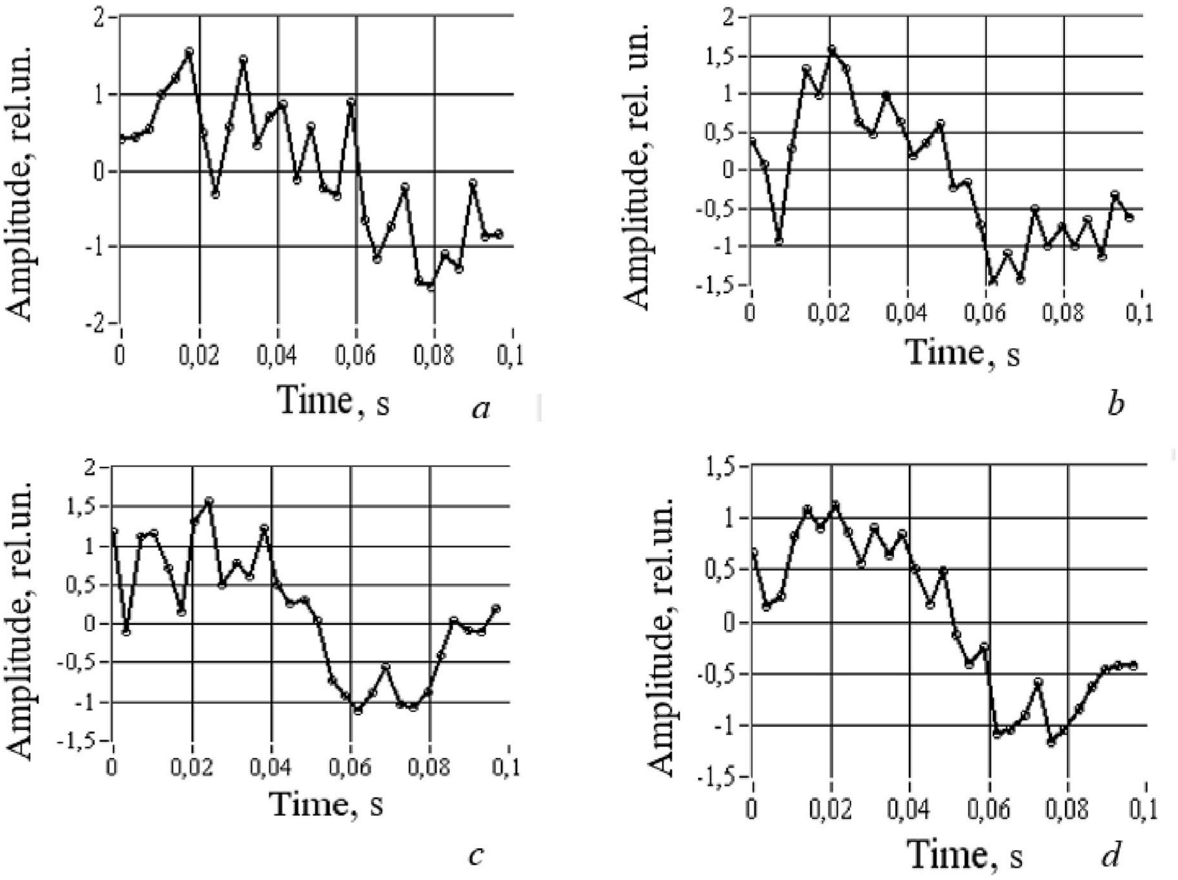

Fig. 7. The first (*a*), second (*b*), third (*c*) realizations of the simulated two-component GGA model at $\sigma_{noise} = 0.5$ and the time-averaged realization (d).

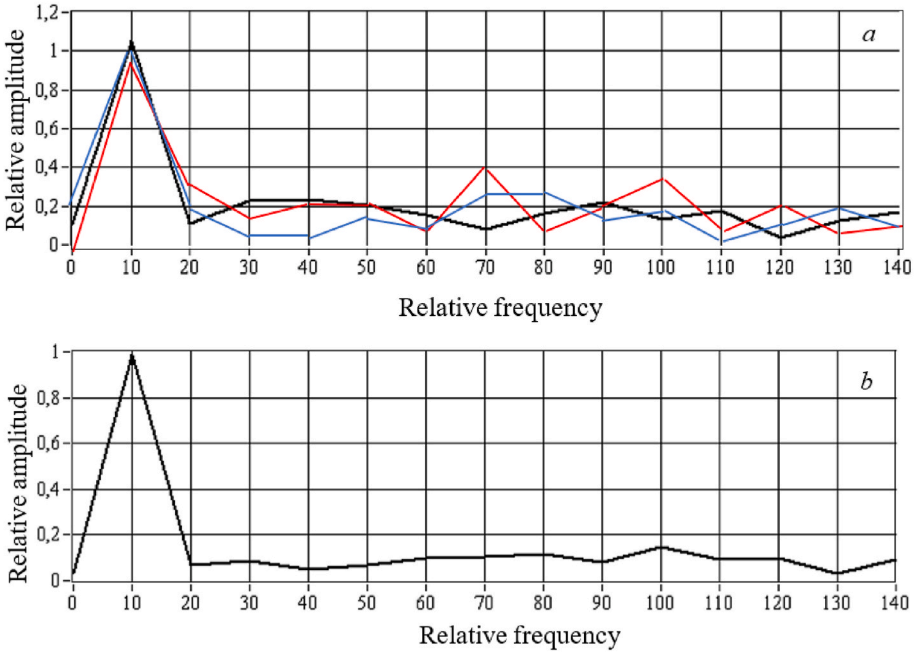

Fig. 8. Spectra of three realizations (a) and the spectrum of the time-averaged realization (b) for the two-component allowance model at $\sigma_{noise} = 0.5$.

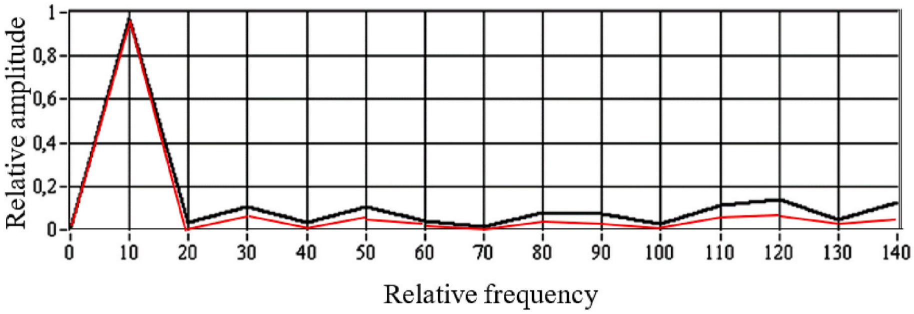

Fig. 9. Time-averaged spectra of realizations for three (black line) and for twenty-five (red line) realizations. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

geometric parameters of the gear (on a measuring station or coordinate measuring machine).

6. Future study

The advantage of the proposed methodology in terms of the added value that its results provide is a significant reduction in labor-intensive experimental studies necessary to solve the actual problem – optimizing the amount of GGA. This problem, i.e., the problem of limiting the amount of GGA is that the amount of GGA should be minimal, but at the same time sufficient to obtain the required gear accuracy. Indeed, the paper discusses a gear grinding strategy, according to which the measurement of the actual GGA before performing the grinding operation was carried out on three (out of thirty) gears, which were selectively taken from a batch of gears under serial production conditions.

In accordance with the method presented in the article, future research could be aimed at accumulating the results of the practical application of this method in production conditions, checking the reliability of the method and determining production conditions for its effective use, for example, the influence of batch size on the accuracy of GGA assessment, etc. As a result, both advantages and drawbacks of the stochastic GGA model for gear grinding will be found out.

7. Conclusions

1. A gear grinding allowance stochastic model created in the article is based on a conception of a random process which is defined by two sample variables: a sample time variable, which is proportional to the angle of rotation of the gear from 0 to 2π, and a sample space variable, which is order number of the gear to be measured. That is why two interpretations of a random process are used. The first interpretation is a time-indexed family of random variables when time is a deterministic value. The second one is a space-indexed family of random variables when space is defined by the ensemble of realizations (in time) of a random process.
2. The proposed stochastic model of the gear grinding allowance is based on the assumption (hypothesis) about the representation of the gear grinding allowance distribution, consisting of two components: a sinusoidal deterministic component with a period of 2π, caused by the eccentricity of the gear to be ground, and a random component caused by the residual deformation (distortion) of the gear due to its chemical-thermal treatment.
3. Based on LabVIEW virtual instruments for the synthesis and transformation of the gear grinding allowance signal, a method of simulation of the gear grinding allowance and the corresponding stochastic gear grinding allowance model are proposed.

4. On the basis of the experimental measurement of the gear grinding allowance for three real gears, the time and frequency (after Fast Fourier Transform) characteristics of the gear grinding allowance signal were obtained.
5. It is shown that the spectrum of the averaged realization for random harmonic components tends to disappear, which corresponds to the following property of a stochastic random process, i.e., random stationary ergodic process: when summing a large number of harmonics with random phases, the average over the set (equal to the time average) disappears due to the mutual compensation of harmonics with opposite signs (or with a phase shift π). In other words, the amplitude spectrum of an ensemble-averaged random process tends to disappear.
6. The theoretical-probabilistic approach can be used to study real gears by simulating a random process using white Gaussian noise. This allows studying the gear grinding allowance on a simulation model using different methods of its alignment in gear grinding, i.e., without using expensive experimental studies of real gears.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Acknowledgements

This research was carried out at the Department of Mechanical, Manufacturing, and Biomedical Engineering of Trinity College of Dublin in accordance with the project “Data driven optimization of complex-shaped parts produced by grinding and abrasive processes”, funded by Enterprise Ireland and the European Commission under the Horizon 2020 Marie Skłodowska-Curie Actions.

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