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The problem of stationary thermoelasticity for a piecewise homogeneous transversely isotropic space under the influence of a heat flux specified at infinity is considered

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Abstract. An exact solution to the problem of thermoelasticity about interfacial circular inclusion, which is in complete adhesion with different transversely isotropic half-spaces, is constructed. Analytical dependencies are obtained between the parameters of the linear displacement of the inclusion and the characteristics of the force and temperature fields. The order of the singularity of surges and displacements near the inclusion is determined. Expressions are obtained for the stress intensity factor at the inclusion boundary, as well as the numerical dependences of these coefficients on the polar angle, temperature, and load.

1. Statement of the problem and its reduction to a system of two-dimensional singular integral equations (SIE)

The problem of stationary thermoelasticity for a piecewise homogeneous transversely isotropic space under the influence of a heat flux $q_0(x_1, x_2, x_3)$ specified at infinity is considered In $x_3 = 0$ plane of connecting two transversely isotropic spaces contained absolutely rigid thermally insulated inclusion is contained which occupies the region Ω: $\{0 \le r \le a, 0 \le \varphi \le 2\pi\}$. The inclusion of an arbitrary applied load, which leads to the resultant force $P = (P_1, P_2, P_3)$ and the main moment $M = (M_1, M_2, M_3)$. The location of the face of inclusion after deformation describe the function

$$\begin{aligned} &\zeta_6^{\pm} = \zeta_6^0 + \vartheta_0^{\pm}(x_1, x_2), \quad \zeta_k^{\pm} = \zeta_k^0, \quad k = 4, 5, \quad (x_1, x_2) \in \Omega, \\ &\zeta_4^0 = \delta_1 - \varphi_3 x_3, \quad \zeta_5^0 = \delta_2 + \varphi_3 x_1, \quad \zeta_6^0 = \delta_3 + \varphi_2 x_2 + \varphi_1 x_2, \end{aligned} \tag{1}$$

where

$$\begin{aligned} \{\zeta_k^{\pm}\}_{k=1}^{8} = \{&\sigma_3(x_1, x_2, \pm 0), \sigma_4(x_1, x_2, \pm 0), \sigma_5(x_1, x_2, \pm 0), \\ &u_1(x_1, x_2, \pm 0), u_2(x_1, x_2, \pm 0), u_3(x_1, x_2, \pm 0), T(x_1, x_2, \pm 0), Q(x_1, x_2, \pm 0)\}, \\ \sigma = \{\sigma_k\}_{k=1}^{6} &= \{\sigma_x, \sigma_y, \sigma_z, \tau_{yz}, \tau_{xz}, \tau_{xy}\}, \quad u = \{u_k\}_{k=1}^{3} = \{u, v, w\}, \end{aligned}$$

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$\vartheta_0^{\pm}(x_1, x_2)$ is setting the form of inclusion, respectively, when $x_3 = \pm 0$, δ_k, φ_k, $k = 1, 2, 3$ are translational movements and turning angles around the corresponding axes, $T(x_1, x_2, x_3)$ is temperature, $Q(x_1, x_2, x_3)$ is heat flow.

The case is considered when the heat flux is set on the inclusion, and the inclusion is fully coupled to half-spaces, in this case the boundary conditions have the form [1,2]:

$$\begin{aligned} &\chi_4^{\pm}(x_1, x_2) = (1 \pm 1)\zeta_4^0, \quad \chi_5^{\pm}(x_1, x_2) = (1 \pm 1)\zeta_5^0, \\ &\chi_6^{\pm}(x_1, x_2) = \vartheta^{\pm}(x_1, x_2) + (1 \pm 1)\zeta_6^0, \vartheta^{\pm} = \vartheta_0^{+} \pm \vartheta_0^{-}, \quad (x_1, x_2) \in \Omega, \end{aligned} \tag{2}$$

$$\zeta_8^{\pm}(x_1, x_2, \pm 0) = q(x_1, x_2), (x_1, x_2) \in \Omega. \tag{3}$$

Given the conditions

$$\begin{aligned} &\chi_k^{-}(x_1, x_2) = 0, \quad k = \widehat{1, 6}, \quad (x_1, x_2) \notin \Omega, \\ &\lambda_3^{+}\partial_2\zeta_7(x_1, x_2, +0) = \lambda_3^{-}\partial_2\zeta_7(x_1, x_2, -0), \quad \zeta_7(x_1, x_2, +0) = \zeta_7(x_1, x_2, -0), \end{aligned} \tag{4}$$

that reflect the fact of the full adhesion of half-spaces outside the inclusion, and using the technique described in [1–9], regarding unknown jumps and temperature surges $\chi_k^{-}(x, y)$ $(k = 1, 2, 3, 7)$,

$$\chi_k^{\pm} = \langle \chi_k(x_1, x_2) \rangle^{\pm} = \zeta_k(x_1, x_2, +0) \pm \zeta_k(x_1, x_2, -0), \quad (x_1, x_2) \in \Omega, \tag{5}$$

the following system of two-dimensional SIE is obtained:

$$\begin{aligned} &\frac{1}{2\pi}\iint_{\Omega}\left\{q_{31}\chi_1^{-}\frac{x_1-\xi_1}{r_0^2} + q_{32}^{-}\chi_2^{-}\partial_2\frac{x_1-\xi_1}{r_0} + \chi_3^{-}\left[q_{32}\partial_1\frac{x_1-\xi_1}{r_0} + \tilde{q}_{21}\partial_2\frac{x_2-\xi_2}{r_0}\right]\right\}d\xi_1\,d\xi_2 = g_1, \\ &\frac{1}{2\pi}\iint_{\Omega}\left\{q_{31}\chi_1^{-}\frac{x_2-\xi_2}{r_0^2} + \chi_2^{-}\left[q_{32}\partial_2\frac{x_2-\xi_2}{r_0} + \hat{q}_{21}\partial_1\frac{x_1-\xi_1}{r_0}\right] + q_{32}^{-}\chi_3^{-}\partial_1\frac{x_2-\xi_2}{r_0}\right\}d\xi_1\,d\xi_2 = g_2, \\ &\frac{1}{2\pi}\iint_{\Omega}\left\{q_{41}\chi_1^{-}\frac{1}{r_0} - q_{42}\left[\chi_2^{-}\frac{x_2-\xi_2}{r_0^2} + \chi_3^{-}\frac{x_1-\xi_1}{r_0^2}\right]\right\}d\xi_1\,d\xi_2 = g_3, \\ &q_{66}\chi_8^{-}(x_1, x_2) - \frac{q_{65}}{2\pi}\iint_{\Omega}\chi_7^{-}(\xi_1, \xi_2)\frac{1}{r_0^3}d\xi_1\,d\xi_2 = \chi_8^{+}, \end{aligned} \tag{6}$$

where

$$\begin{aligned} &g_1(x_1, x_2) = \chi_4^{+}(x_1, x_2) + \frac{q_{34}}{2\pi}\iint_{\Omega}\chi_6^{-}\partial_1\frac{1}{r_0}d\xi_1\,d\xi_2 - \frac{q_{35}}{2\pi}\iint_{\Omega}\chi_7^{-}\frac{x_1-\xi_1}{r_0^2}d\xi_1\,d\xi_2, \\ &g_2(x_1, x_2) = \chi_5^{+} + \frac{q_{34}}{2\pi}\iint_{\Omega}\chi_6^{-}\partial_2\frac{1}{r_0}d\xi_1\,d\xi_2 - \frac{q_{35}}{2\pi}\iint_{\Omega}\chi_7^{-}\frac{x_2-\xi_2}{r_0^2}d\xi_1\,d\xi_2, \\ &g_3(x_1, x_2) = \chi_6^{+}(x_1, x_2) - q_{44}\chi_6^{-}(x_1, x_2) - \frac{q_{45}}{2\pi}\iint_{\Omega}\chi_7^{-}\frac{1}{r_0}d\xi_1\,d\xi_2, \\ &r_0 = \sqrt{(x_1-\xi_1)^2 + (x_2-\xi_2)^2}, \quad \partial_1 \equiv \frac{\partial}{\partial x}, \quad \partial_2 \equiv \frac{\partial}{\partial y}. \end{aligned}$$

The quantities δ_k, φ_k, $k = 1, 2, 3$ are determined from the following equilibrium equations

$$\begin{aligned} &\iint_{\Omega}\chi_k^{-}(x_1, x_2)\,dx_1\,dx_2 = P_{4-k}, \quad k = 1, 2, 3, \\ &\iint_{\Omega}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\chi_1^{-}(x_1, x_2)\,dx_1\,dx_2 = \begin{pmatrix} M_1 \\ M_2 \end{pmatrix}, \quad \iint_{\Omega}[x_1\chi_2^{-}(x_1, x_2) - x_2\chi_3^{-}(x_1, x_2)]\,dx_1\,dx_2 = M_3, \end{aligned} \tag{7}$$

System (6) relatively new unknowns: $\tau^\pm = \chi_3^\pm + i\chi_2^\pm$, $u^\pm = \chi_4^\pm + i\chi_5^\pm$, χ_7^- easily reduces to the following two-dimensional system SIE:

$$\begin{aligned}
&q_{31}\bar{K}_0[\chi_1^-] + \frac{q_{32}^+}{2}K[\tau^-] + \frac{q_{32}^-}{2}\mathrm{D}K[\vartheta\bar{\tau}^-] = u^+ + q_{34}\mathrm{D}K[\chi_6^-] - q_{35}\bar{K}_0[\chi_7^-],\\
&q_{31}K_0[\chi_1^-] + \frac{q_{32}^+}{2}K[\bar{\tau}^-] + \frac{q_{32}^-}{2}\bar{\mathrm{D}}K[\bar{\vartheta}\tau^-] = \bar{u}^+ + q_{34}\bar{\mathrm{D}}K[\chi_6^-] - q_{35}K_0[\chi_7^-],\\
&q_{41}K[\chi_1^-] + \frac{q_{42}}{2}[K_0[\tau^-] + \bar{K}_0[\bar{\tau}^-]] = \chi_6^+ - q_{44}\chi_6^- - q_{45}K[\chi_7^-],\\
&-\frac{q_{65}}{2\pi}\mathrm{D}\bar{\mathrm{D}}K[\chi_7^-] = \chi_8^+(x,y),
\end{aligned} \tag{8}$$

where

$$\begin{aligned}
&K[\chi_j^-] = \frac{1}{2\pi}\iint_\Omega \frac{\chi_j^-(t,\tau)}{r_0}dt\,d\tau, \quad K_0[\chi_j^-] = \frac{1}{2\pi}\iint_\Omega \frac{\chi_j^-(t,\tau)}{\vartheta}dt\,d\tau,\\
&\mathrm{D} = \partial_1 + i\partial_2, \quad \vartheta = (x-t) + i(y-\tau), \quad r_0^2 = \vartheta\bar{\vartheta}.
\end{aligned}$$

2. Two-dimensional system solution SIE

We obtain the exact solution to system (8) using the approach described in [3–7]. For definiteness, we will assume that the heat flux in the region Ω changes linearly:

$$q(x,y) = q_0(b_{00} + b_{10}x + b_{01}y). \tag{9}$$

Considering that the inclusion occupies a circular area, move on to the thermoelastic characteristics of space in cylindrical coordinates (ρ,φ,z), and denote the jumps and sums when passing through the plane $z=0$ so

$$\{\hat{v}_k^\pm(\rho,\varphi)\}^8 = \{\langle\sigma_z\rangle^\pm, \langle\tau_{z\varphi}\rangle^\pm, \langle\tau_{z\rho}\rangle^\pm, \langle u_\rho\rangle^\pm, \langle u_\varphi\rangle^\pm, \langle w\rangle^\pm, \langle T\rangle^\pm, \langle q\rangle^\pm\}. \tag{10}$$

We introduce combinations of displacements and stresses: $\tilde{u}^\pm = \tilde{v}_4^\pm + i\tilde{v}_5^\pm$, $\tilde{\tau}^\pm = \tilde{v}_3^\pm + i\tilde{v}_2^\pm$, then given the ratios: $u^\pm(\rho\cos\varphi,\rho\sin\varphi) = \tilde{u}^\pm(\rho,\varphi)e^{i\varphi}$, $\tau^\pm(\rho\cos\varphi,\rho\sin\varphi) = \tilde{\tau}^\pm(\rho,\varphi)e^{i\varphi}$, move on to new unknown functions:

$$\begin{aligned}
&v_1^-(r,\varphi) = \chi_1^-(\rho\cos\varphi,\rho\sin\varphi), \quad v_2^\pm = e^{-i\varphi}\tau^\pm(\rho\cos\varphi,\rho\sin\varphi),\\
&v_3(r,\varphi) = e^{-i\varphi}u^-(\rho\cos\varphi,\rho\sin\varphi), \quad v_5^-(r,\varphi) = \chi_7^-(\rho\cos\varphi,\rho\sin\varphi).
\end{aligned} \tag{11}$$

Recent sought in the form $(j = 2,3,5)$

$$v_j^\pm(\rho,\varphi) = \sum_{n=-\infty}^{\infty}\mathrm{V}_n^{j,\pm}(\rho)e^{in\varphi}, \quad \mathrm{V}_n^{j,-}(\rho) = \Phi_n[v_j^-] \equiv \frac{1}{2\pi}\int_{-\pi}^{\pi} v_j^-(\rho,\varphi)e^{-in\varphi}\,d\varphi. \tag{12}$$

We pass to the polar coordinates in system (8) and apply the finite Fourier transform. Then, given the formulas [2]

$$\begin{aligned}
&\frac{1}{\sqrt{\rho^2 - 2\rho\rho_*\cos(\varphi-\varsigma) + \rho_*^2}} = \sum_{n=-\infty}^{\infty} e^{ik(\varphi-\varsigma)}\int_0^\infty J_k(t\rho)J_k(t\rho_*)\,dt,\\
&\int_{-\pi}^{\pi}\frac{e^{-i\varphi m}\,d\varphi}{\rho-\rho_* e^{\pm i\varphi}} = \pm\mathrm{W}_{m\pm1,m}^0(\rho,\rho_*), \quad \mathrm{W}_{kn}^0(\rho,\rho_*) = \int_0^\infty J_k(\rho t)J_n(\rho_* t)\,dt,
\end{aligned}$$

representation (12), and the convolution theorem, after simple transformations, we obtain with respect to $\mathrm{V}_n^{j,\pm}(\rho)$ the following system of integral equations with Weber-Sonin kernels ($n = 0, \pm 1, \pm 2, \ldots,\ 0 < \rho < a$)

$$\begin{aligned}
&2q_{31}\mathrm{W}_{n+1,n}[\mathrm{V}_n^{1,-}] - q_{32}\mathrm{W}_{n+1,n+1}[\mathrm{V}_n^{2,-}] + q_{32}^{+}\mathrm{W}_{n+1,n-1}[\bar{\mathrm{V}}_{-n}^{2,-}] = 2Q_{1n},\\
&-2q_{31}\mathrm{W}_{n-1,n}[\mathrm{V}_n^{1,-}] - q_{32}\mathrm{W}_{n-1,n-1}[\bar{\mathrm{V}}_{-n}^{2,-}] + q_{32}^{+}\mathrm{W}_{n-1,n+1}[\mathrm{V}_n^{2,-}] = 2Q_{2n},\\
&2q_{41}\mathrm{W}_{n,n}[\mathrm{V}_n^{1,-}] - q_{42}\mathrm{W}_{n,n+1}[V_n^{2,-}] + q_{42}\mathrm{W}_{n,n-1}[\bar{\mathrm{V}}_{-n}^{2,-}] = 2Q_{3n},
\end{aligned} \tag{13}$$

$$\mathrm{W}_{n+1,n+1}[\tilde{V}_n^{5,-}] = Q_{4n}(\rho), \tag{14}$$

where

$$\begin{aligned}
&Q_{1n} = 2\Delta_{12}\delta_{-1,n} + i2\varphi_3\rho\delta_{0,n} - q_{34}\mathrm{W}_{n+1,n+1}[\tilde{\Theta}_n^-] - q_{35}\mathrm{W}_{n+1,n}[\mathrm{V}_n^{5,-}],\\
&Q_{2n} = 2\bar{\Delta}_{12}\delta_{1,n} - i2\varphi_3\rho\delta_{0,n} + q_{34}\mathrm{W}_{n-1,n+1}[\tilde{\Theta}_n^-] + q_{35}\mathrm{W}_{n-1,n}[\mathrm{V}_n^{5,-}],\\
&Q_{3n} = q_{44}\Theta_n^-(\rho) - \Theta_n^+(\rho) - 2\delta_3\delta_{n,0} + \varphi_y\rho(\delta_{n,1} + \delta_{n,-1}) - i\varphi_x\rho(\delta_{n,1} - \delta_{n,-1}) - q_{45}\mathrm{W}_{n,n}[\mathrm{V}_n^{5,-}],\\
&Q_{4n} = \frac{q_0}{q_{65}}\left\{\frac{b_{00}}{2}\rho\delta_{0n} + \frac{\rho^2}{4}[b_{10}(\delta_{1,n} + \delta_{-1,n}) - ib_{01}(\delta_{1,n} - \delta_{-1,n})]\right\},\\
&\Theta_n^{\pm}(\rho) = \Phi_n[\vartheta^{\pm}], \quad \tilde{\Theta}_n^{\pm}(\rho) = \rho^n[\rho^{-n}\Theta_n^{\pm}(\rho)]', \quad \phi_{12} = \varphi_2 + i\varphi_1,\\
&\mathrm{W}_{nk}[f_*] = \int_0^a f_*(\rho_*)\mathrm{W}_{nk}^0(\rho,\rho_*)\rho_*\,d\rho_*, \quad \mathrm{W}_{nk}^0(\rho,\rho_*) = \int_0^\infty J_n(t\rho)J_k(t\rho)\,dt.
\end{aligned}$$

For the operator $\mathrm{W}_{l,l}$ there exists the converse [2, 3], which is an isomorphism into and admits a representation

$$\mathrm{W}_{l,l}^*[f(\rho)] = -\frac{2}{\pi\rho^{1-l}}\partial_\rho\int_\rho^a \frac{t^{1-2l}\,dt}{(t^2-\rho^2)^{1/2}}\frac{d}{dt}\int_0^t \frac{\tau^{1+l}f(\tau)}{(t^2-\tau^2)^{1/2}}d\tau.$$

Using which and decomposition (12), and also considering $\mathrm{V}_n^{5,-}(\rho) = \rho^n\int_\rho^a \rho^{-n}\tilde{\mathrm{V}}_n^{5,-}(\rho)\,d\rho$, from (14) we obtain

$$\begin{aligned}
&\mathrm{V}_n^{5,-}(\rho) = \frac{q_0}{q_{65}}\left\{2b_{00}\sqrt{a^2-\rho^2}\delta_{0,n} + \frac{4}{3}\rho\sqrt{a^2-\rho^2}[b_{10}(\delta_{1,n} + \delta_{-1,n}) - ib_{01}(\delta_{1,n} - \delta_{-1,n})]\right\},\\
&v_5^-(\rho,\varphi) = \frac{q_0}{q_{65}}\left[2b_{00}\sqrt{a^2-\rho^2} + \frac{8}{3}(b_{10}\cos\varphi + b_{01}\sin\varphi)\rho\sqrt{a^2-\rho^2}\right].
\end{aligned}$$

Applying to the solution of system (13), the approach described in [3–7], are obtain explicit expressions for the stresses jumps on the inclusion in the case of an axisymmetric inclusion $\vartheta_0^{\pm}(x,y) = \vartheta_0(x^2+y^2)$, $\Theta_n^{\pm}(\rho) = \Phi_n[\vartheta^{\pm}] = \delta_{0,n}\Theta^{\pm}(\rho)$:

$$\begin{aligned}
\langle\sigma_z\rangle^- = &-\frac{1}{\rho}\left\{\frac{1}{p}i\int_\rho^a \frac{t[\varphi_{10}(t) + \varphi_{10}^*(t)]\,dt}{\sqrt{t^2-\rho^2}}\right\}'\\
&- e^{i\varphi}\left\{\frac{1}{\pi}\int_\rho^a \frac{t[\varphi_{11}(t) + \varphi_{11}^*(t)]\,dt}{\sqrt{t^2-\rho^2}}\right\}' - e^{-i\varphi}\left\{\frac{1}{\pi}\int_\rho^a \frac{t[\bar{\varphi}_{11}(t) + \bar{\varphi}_{11}^*(t)]\,dt}{\sqrt{t^2-\rho^2}}\right\},\\
\langle\tau_{z\varphi}\rangle^- + \langle\tau_{z\rho}\rangle^- = &\ v_2(\rho,\varphi) = -\left\{\frac{1}{\pi}\int_\rho^a \frac{[\varphi_{20}(t)+\varphi_{30}(t)+\varphi_{20}^*(t)]\,dt}{\sqrt{t^2-\rho^2}}\right\}'\\
&+ \rho e^{i\varphi}\left\{\frac{1}{\pi\rho^2}\int_\rho^a \frac{t[\varphi_{21}(t)+\varphi_{31}(t)+\varphi_{21}^*(t)]\,dt}{\sqrt{t^2-\rho^2}}\right\}' - \frac{e^{-i\varphi}}{\rho}\left\{\frac{1}{\pi}\int_\rho^a \frac{t[\bar{\varphi}_{21}(t)-\bar{\varphi}_{31}(t)+\bar{\varphi}_{31}^*(t)]\,dt}{\sqrt{t^2-\rho^2}}\right\}'.
\end{aligned} \tag{15}$$

Here we use the notation:

$$\varphi_{10} = -\frac{2\delta_z}{b_2\sqrt{\kappa_*}}\mathrm{Re}\,\omega_1\rho + \frac{1}{b_2}\mathrm{Re}\,\Gamma_1[\partial_\rho \Im_0[\Theta^*]] + \frac{\lambda_0 q_{34}}{2a_1}\mathrm{Im}\,\Gamma_1[\partial_\rho\Theta^-],$$
$$\Theta^* = q_{44}\Theta^-(\rho) - \Theta^+(\rho), \quad \varphi_{30}^- = -\frac{i8\phi_z}{b_3}\rho, \quad \varphi_{31}^- = -\frac{\delta_{xy}}{b_3},$$
$$\varphi_{20} = -\frac{2\delta_z\lambda_0}{a_2\sqrt{\kappa_*}}\mathrm{Im}\,\omega_1(\rho) + \frac{\lambda_0}{a_2}\mathrm{Im}\,\Gamma_1[\partial_\rho \Im_0[\Theta^*]] - \frac{q_{34}}{b_1}\mathrm{Re}\,\Gamma_1[\partial_\rho\Theta^-],$$
$$\varphi_{10} = -\frac{2\delta_z}{b_2\sqrt{\kappa_*}}\mathrm{Re}\,\omega_1(\rho) + \frac{1}{b_2}\mathrm{Re}\,\Gamma_1[\partial_\rho \Im_0[\Theta^*]] + \frac{\lambda_0 q_{34}}{2a_1}\mathrm{Im}\,\Gamma_1[\partial_\rho\Theta^-],$$
$$\varphi_{11} = \frac{2\bar{\phi}_{yx}}{b_2\sqrt{\kappa_*}}\mathrm{Re}\{(\rho+2a\gamma_1)\omega_1(\rho)\} - \frac{\lambda_0\bar{\delta}_{xy}}{a_1\sqrt{\kappa_*}}\mathrm{Im}\,\omega_1(\rho),$$
$$\varphi_{21} = \frac{2\lambda_0\bar{\phi}_{yx}}{a_2\sqrt{\kappa_*}}\mathrm{Im}\{(\rho+2a\gamma_1)\omega_1\rho\} - \frac{\bar{\delta}_{xy}}{b_1\sqrt{\kappa_*}}\mathrm{Re}\,\omega_1\rho,$$
$$\varphi^*_{1j-1}(\rho) = \frac{1}{b_2}\mathrm{Re}\,R_1[f^t_{2j}(\rho)] - \frac{\lambda_0}{a_1}\mathrm{Im}\,R_1[f^t_{1j}(\rho)],$$
$$\varphi^*_{2j-1}(\rho) = \frac{\lambda_0}{a_2}\mathrm{Im}\,R_1[f^t_{2j}(\rho)] + \frac{1}{2b_1}\mathrm{Re}\,R_1[f^t_{1j}(\rho)], \quad j=1,2$$
$$f^t_{22}(\rho) = -q_0\frac{q_{45}}{q_{65}}\frac{4}{3}(b_{10}-ib_{01})\frac{\sqrt{\pi}}{2\sqrt{2}}\rho(a^2-\rho^2),$$
$$f^t_{12}(\rho) = q_0\frac{q_{35}}{q_{65}}(b_{10}-ib_{01})\frac{2\sqrt{2}}{3\sqrt{\pi}}\left[\frac{4}{3}a^3 + (a^2-\rho^2)\ln\frac{a-\rho}{a+\rho} - 2\rho a\right],$$
$$f^t_{21}(\rho) = -q_0\frac{q_{45}}{q_{65}}b_{00}\sqrt{\frac{\pi}{2}}(a^2-\rho^2), \quad f^t_{11}(\rho) = q_0\frac{q_{35}}{q_{65}}\frac{b_{00}}{\sqrt{2\pi}}\left[(a^2-\rho^2)\ln\frac{a-\rho}{a+\rho} - 2\rho a\right],$$
$$\Gamma_j[f(\rho)] = \frac{1}{\kappa_*}\left[f(\rho) - \omega_j(\rho)\frac{\lambda_j}{\pi}\int_{-a}^{a}\frac{f(t)}{\omega_j(t)}\frac{dt}{t-\rho}\right], \quad \omega_j(\rho) = \left(\frac{a+\rho}{a-\rho}\right)^{\gamma_j}, \quad \kappa_* = 1-\lambda_0^2,$$
$$\lambda_0^2 = \kappa_1\kappa_2, \quad \kappa_j = \frac{a_j}{b_j}, \quad \lambda_j = (-1)^{j+1}i\lambda_0, \quad \gamma_j = (-1)^{j+1}i\alpha_0, \quad \alpha_0 = \frac{1}{2\pi}\ln\frac{1+\lambda_0}{1-\lambda_0}, \quad l=1,2.$$

For δ_k, φ_k, $k=1,2,3$, are obtained the following expressions:

$$\delta_x = m_4\frac{P_1}{a} + m_5\frac{M_2}{a^2} - q_0 b_{10} m_1^-, \quad \delta_y = m_4\frac{P_2}{a} - m_5\frac{M_1}{a^2} + q_0 b_{10} m_1^+,$$
$$\delta_z = m_1\frac{P_1}{2a} + \frac{h}{a}(m_2+m_3) - \frac{q_0 b_{00}}{2a^{-2}}m_1(m_8+m_9), \quad \varphi_z = \frac{3b_3 M_3}{2\pi a^3},$$
$$\varphi_y = m_6\frac{P_1}{a^2} + m_7\frac{M_2}{a^3} + \frac{q_0 b_{10}}{a}m_2^-, \quad \varphi_x = m_6\frac{P_2}{a^2} - m_7\frac{M_1}{a^3} - \frac{q_0 b_{10}}{a}m_2^+,$$

where

$$
\begin{aligned}
&m_4 = -\frac{l_{11}n_{21}}{\Delta}, \quad m_5 = \frac{l_{21}n_{11}}{\Delta}, \quad m_1^{\pm} = \frac{l_{22}n_{11} \pm l_{12}n_{21}}{\Delta}, \\
&m_6 = \frac{l_{11}n_{22}}{\Delta}, \quad m_7 = -\frac{l_{21}n_{12}}{\Delta}, \quad m_2^{\pm} = \frac{l_{22}n_{12} \pm l_{12}n_{22}}{\Delta}, \\
&n_{11} = \frac{a_1}{b_2}\frac{\alpha_0}{\lambda_0}, \quad n_{12} = 1 + \frac{b_1}{b_3}\frac{\lambda_0}{\alpha_0}, \quad l_{11} = \frac{b_1}{2\pi}\frac{\lambda_0}{\alpha_0}, \quad l_{12} = l_{11}a^3 m_{13}, \\
&n_{21} = \frac{2}{3}(1 - 8\alpha_0^2), \quad n_{22} = -\frac{\alpha_0}{\lambda_0}\frac{a_2}{b_1}, \quad l_{21} = \frac{b_2}{4\pi}\frac{\lambda_0}{\alpha_0}, \quad l_{22} = l_{21}2a^2(m_{14} - m_{15}), \\
&m_{12} = \frac{q_{45}}{q_{65}}\frac{(2\pi)^{3/2}}{9}\frac{\sqrt{\kappa_*}}{a_2}(1+\alpha_0^2)\alpha_0^2, \quad m_{13} = \frac{\sqrt{\kappa_*}}{b_1}\frac{q_{35}}{q_{65}}\frac{16\sqrt{2\pi}}{9}\frac{\alpha_0}{\lambda_0}, \\
&m_{10} = \frac{1}{b_2}\frac{q_{45}}{q_{65}}\frac{4\pi\sqrt{2\pi}}{9a^{-1}}\frac{\sqrt{\kappa_*}}{\lambda_0}(1+\alpha_0^2)\alpha_0(1-a), \\
&m_{11} = \frac{q_{35}}{q_{65}}\frac{2\sqrt{2}}{3\sqrt{\pi}}\left\{\frac{\alpha_0\sqrt{\kappa_*}}{a_1}\frac{(1+\alpha_0^2)\pi}{(2a)^{-3}\Gamma(4)}\left[\frac{\pi}{\lambda_0} - \frac{3\alpha_0^2+1}{\alpha_0(1+\alpha_0^2)}\right] + \frac{\alpha_0\sqrt{\kappa_*}}{2}\right\}, \\
&m_{14} = \frac{q_{35}}{q_{65}}\frac{8\sqrt{\pi 2}}{45}\frac{\tau_1\sqrt{\kappa_*}}{\lambda_1 b_2}(1+\alpha_0^2)a(1+4\alpha_0^2)\alpha_0, \\
&m_{15} = \frac{q_{35}}{q_{65}}\frac{4\sqrt{2\pi}}{9}\frac{\sqrt{\kappa_*}}{a_1}\left\{\alpha_0(1+\alpha_0^2)(2+i\alpha_0)\left[\frac{\pi}{\lambda_0} - \frac{3\alpha_0^2+1}{\alpha_0(1+\alpha_0^2)}\right] - 4a\alpha_0^2\right\}.
\end{aligned}
$$

3. Calculation of generalized intensity factors and analysis of the results

To determine the features of the normal stress field in the vicinity of the inclusion, we obtain the expression for normal stresses in the plane of connection of two transversely isotropic half-spaces for $\rho > a$. For this we use the singular integral relations, of the work [1], and solutions (15) get

$$
\begin{aligned}
\sigma_z\big|_{z=0} = q_{11}v_1^- + q_{15}v_5^- - \frac{q_{12}}{2\rho}\Bigg\{ &\cos\varphi \mathrm{Re}\left[\int_0^a 2\varphi_{21}^*(t)\,dt + \int_0^a \frac{\varphi_{21}^*(t)}{(\rho^2-t^2)^{3/2}}dt\right] \\
- 2\rho\int_0^a \frac{t\varphi_{20}^*(t)}{(\rho^2-t^2)^{3/2}}dt &+ \sin\varphi \mathrm{Im}\left[\int_0^a 2\varphi_{21}^*(t)\,dt + \int_0^a \frac{\varphi_{21}^*(t)}{(\rho^2-t^2)^{3/2}}dt\right]\Bigg\} - q_{14}\frac{2h\pi}{a}.
\end{aligned}
$$

Using the technique described in [5], we obtain the asymptotic representation for $\rho \to a+0$ for normal stresses, in the case of a given heat flux on inclusion

$$
\sigma_z\big|_{z=0} \cong \frac{\pi}{2}\frac{q_0}{(2a)^{1/2}\sqrt{\rho-a}}\{\gamma_{11}\omega_c^a + \gamma_{12}\omega_s^a\} \tag{16}
$$

where

$$
\begin{aligned}
\gamma_{11} = &-\frac{1}{2b_1}\frac{\mu_{11}}{\sqrt{\kappa_*}}\frac{2a^2}{\lambda_0}(\lambda_0 - \pi\alpha_0) - q_{12}2a^2(b_{10}\cos\varphi - b_{01}\sin\varphi)\left[\frac{\mu_{22}}{\sqrt{\kappa_*}}\frac{2a}{3}\alpha_0^2 + \frac{\mu_{12}}{\sqrt{\kappa_*}}\left(\pi\frac{\alpha_0}{\lambda_0} + \frac{4a}{3}\alpha_0^2\right)\right], \\
\gamma_{12} = &\frac{\lambda_0}{a_2}\frac{\mu_{21}}{\sqrt{\kappa_*}} - \frac{1}{2b_1}\frac{\mu_{11}}{\sqrt{\kappa_*}}\frac{2a^2}{\lambda_0}\alpha_0(\lambda_0 + \pi\alpha_0) \\
&- q_{12}(b_{10}\cos\varphi - b_{01}\sin\varphi)\left\{-\frac{\mu_{22}}{\sqrt{\kappa_*}}\frac{4a^3}{3}\alpha_0(1-\alpha_0^2) + \frac{\mu_{12}}{\sqrt{\kappa_*}}2a^2\left[\pi\frac{\alpha_0^2}{\lambda_0} - \frac{4}{3}a^2\alpha_0^2(1-\alpha_0^2)\right]\right\}, \\
\mu_{11} = &\frac{q_{35}}{q_{65}}\frac{b_{00}}{\sqrt{2\pi}}, \quad \mu_{12} = \frac{q_{45}}{q_{65}}b_{00}\sqrt{\frac{\pi}{2}}, \quad \mu_{22} = \frac{q_{45}}{q_{65}}\frac{\sqrt{2\pi}}{3}, \quad \mu_{21} = \frac{q_{35}}{q_{65}}\frac{2\sqrt{2}}{3\sqrt{\pi}}.
\end{aligned}
$$

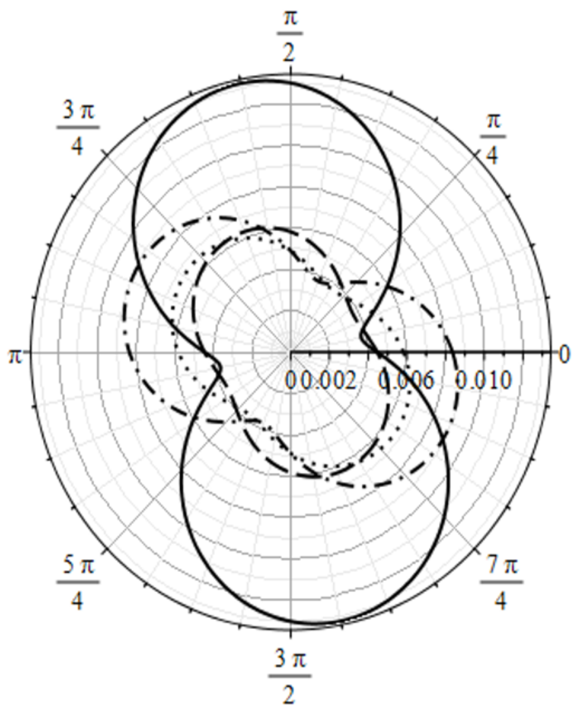

Figure 1. Cadmium/Magnesium

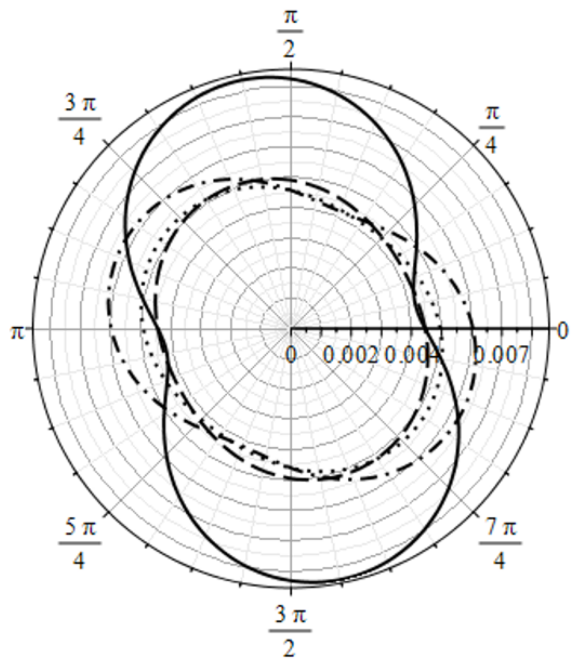

Figure 2. Zn/Al2O3

Following [5], we introduce the generalized stress intensity factor SIF related to q_0, so

$$K_z^* = \sum_{k=1}^{2} \sqrt{\gamma_{1k}^2}$$

Figures 1–4 show the dependencies of the generalized SIF K_z^* on the polar angle for various values of sets of dimensionless parameters (b_{00}, b_{10}, b_{01}) from expression (9) for the heat flow. In all figures dash dot line corresponds to values $(1, 1/3, 1/7)$, dot line corresponds to values $(1, 1/5, 1/7)$, dash line corresponds to values $(1, 1/9, 1/5)$, and solid line corresponds to values $(1, 1/9, 5/9)$. Figure 1 shows the calculations for a combination of transversally isotropic materials Cadmium/Magnesium, in Figure 2 — for the combination Zn-Al_2O_3, in Figure 3 — for the combination Al_2O_3/Cadmium, in Figure 4 — for the combination Zn/Magnesium.

Calculations show that the parameters of the heat flux significantly affect the more generalized SIF for normal stresses. The difference in the properties of transversely isotropic materials also has a significant effect. In all cases, the values of the generalized SIFs significantly depend on the polar angle, and for each combination of parameters the directions of the highest stress concentration in the vicinity of the inclusion are easily determined.

Analytical dependencies are obtained between the parameters of the linear displacement of the inclusion and the characteristics of the force and temperature fields. The order of the singularity of stress jumps and displacements is determined. Expressions are obtained for the stress intensity factor at the inclusion boundary, as well as the numerical dependencies of these coefficients on the polar angle, temperature and loads. A number of interesting patterns of the distribution of stress concentration in the vicinity of the inclusion were found

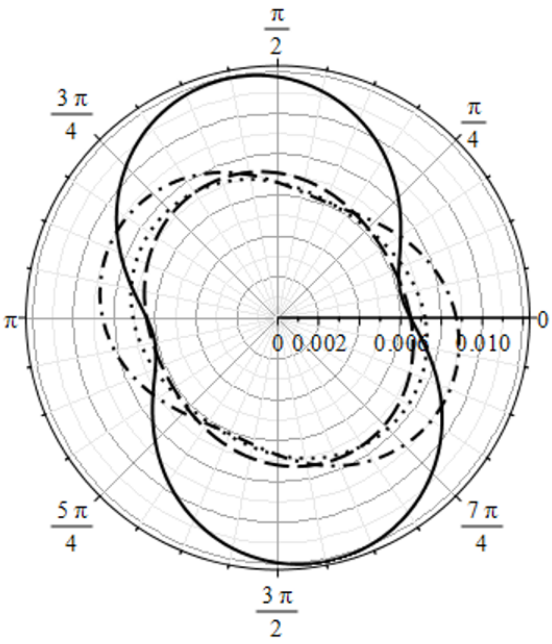

Figure 3. Al2O3/Cadmium

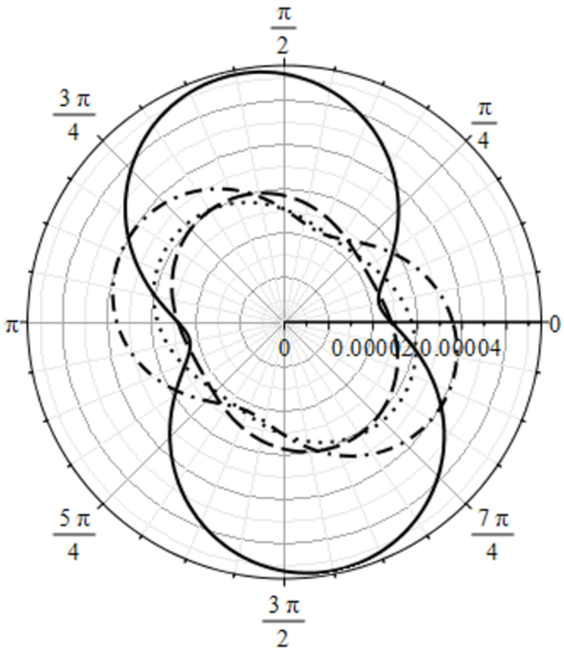

Figure 4. Zn/Magnesium

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