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V. S. Sytnikov, ScD
T. V. Sytnikov, D. A. Tymokhin

ANALYSIS OF PHASE AND FREQUENCY CHARACTERISTIC OF FIRST ORDER COMPONENT FOR THE TASK OF FILTERING AND CONTROL OF SPECIALIZED COMPUTER SYSTEM

Abstract. *The analysis of phase-frequency response of the first order components on an example of Butterworth and Chebyshev digital filters is carried out in the article, changes of these characteristics at variations of filter transfer function coefficients are shown. Dependences of a phase on level of pulsations are received at the set cut-off frequency for the first order low-pass and high-pass Chebyshev filter.*

Keywords*: digital filter, Chebyshev filter, Butterworth filter, frequency response, phase, phase-frequency response, linearization, low-pass filter, high-pass filters, stopband, pass band, quasilinearity*

В. С. Ситников, д-р техн. наук,
Т. В. Ситников, Д. А. Тимохин

АНАЛИЗ ФАЗО-ЧАСТОТНОЙ ХАРАКТЕРИСТИКИ КОМПОНЕНТЫ ПЕРВОГО ПОРЯДКА ДЛЯ ЗАДАЧИ ФИЛЬТРАЦИИ И УПРАВЛЕНИЯ СПЕЦИАЛИЗИРОВАННОЙ КОМПЬЮТЕРНОЙ СИСТЕМЫ

Аннотация. *Проведен анализ фазо-частотных характеристик компонент первого порядка на примере цифровых фильтров Баттерворта и Чебышева, показаны изменения этих характеристик при вариациях коэффициентов передаточной функции фильтра. Получены зависимости фазы от уровня пульсаций при заданной частоте среза для фильтра нижних и верхних частот Чебышева первого рода.*

Ключевые слова*: цифровой фильтр, фильтр Чебышева, фильтр Баттерворта, фаза, фазо-частотная характеристика, линеаризация, фильтр нижних частот, фильтр верхних частот, полоса задержания, полоса пропускания, квазилинейность*

В. С. Ситніков, д-р техн. наук,
Т. В. Ситніков, Д. О. Тимохін

АНАЛІЗ ФАЗО-ЧАСТОТНОЇ ХАРАКТЕРИСТИКИ КОМПОНЕНТИ ПЕРШОГО ПОРЯДКУ ДЛЯ ЗАВДАННЯ ФІЛЬТРАЦІЇ І УПРАВЛІННЯ СПЕЦІАЛІЗОВАНОЇ КОМП'ЮТЕРНОЇ СИСТЕМИ

Анотація. *Проведено аналіз фазо-частотних характеристик компонент першого порядку на прикладі цифрових фільтрів Баттерворта і Чебишева, показані зміни цих характеристик при варіаціях коефіцієнтів передавальної функції фільтра. Отримано залежності фази від рівня пульсацій при заданій частоті зрізу для фільтра нижніх і верхніх частот Чебишева першого роду.*

Ключові слова*: цифровий фільтр, фільтр Чебишева, фільтр Баттерворта, фаза, фазо-частотна характеристика, лінеаризація, фільтр нижніх частот, фільтр верхніх частот, смуга затримання, смуга пропускання, квазілінійність*

When designing the complex control and collecting data from sensors systems, for example, for nuclear power plants and related to them industries, appears task of correction and adjustment of parameters of frequency-dependent components which are part of composition of such specialized computer systems. Such systems are a combination of hardware and software with sophisticated architecture and a variety of connections. The specialized computer system executes plenty of tasks, one of which is a task of control and correction of system characteristics depending on operating conditions and environmental changes, changes in the technological process and changes in the system itself [1–11].

For the design of these components it is necessary to solve task of the analysis of transfer function coefficients of the processing path component influence on the properties of amplitude and phase-frequency response (PFR). Control of the frequency response function (FRF) properties was considered in the article [12] and possibility of both separate and complex control was shown in the article [13]. Such task is typical for adaptive and reconfigurable devices, including filters which have identical mathematical description. [1–4].

The importance of the phase-frequency characteristics of the component comes out of the necessity of the pretreatment of signals from sensors without phase distortion, which leads to additional errors in the processing path. For these purposes, non-recursive filters with linear phase-frequency characteristic were used earlier, but the type and speed of roll-off of AFC hinders their widespread use. The use of recursive filters is only limited by the use of Bessel filter [6, 7]. That's why the problem of determining of the conditions appears, when linearization or quasi-linearization PFC recursive filters is possible.

Let's consider widely used typical digital filters as components of pre-processing and filtration path. It is known that for ease of configuration and control high-order components are designed based on the components of the first and second order [13]. Therefore, analysis of the influence of digital filter transfer function coefficients on the properties of its characteristics done by the transfer function of the first order.

Let's consider the PFR properties of first order digital filters and its changes at variations of filter transfer function coefficients. Analysis of the digital filter transfer function coefficients influence on the properties of its characteristics conducted by a first-order transfer function of the form

$$H(z)=\frac{a_0+a_1z^{-1}}{1+bz^{-1}}, \qquad (1)$$

where a_0, a_1, b – real coefficients of the numerator and denominator respectively.

In this case, in general form for low-pass filter (LPF) corresponds transfer function (1) with numerator coefficient $a_1>0$, and for high-pass filter (HPF) – $a_1<0$. It should be noted, than numerator coefficients of the normalized first order digital filters are equal, i.e.

$$a_0=|a_1|,\ a_0>0.$$

Then the transfer function (1) can be written as

$$H(z)=a_0\frac{1\pm z^{-1}}{1+bz^{-1}},$$

were"+" – in the numerator corresponds to LPF, and "–" – HPF.

Through substitution $z^{-1}=\mathrm{e}^{-\mathrm{j}\bar{\omega}}$ and Euler's formula $z^{-1}=\cos\bar{\omega}-\mathrm{j}\sin\bar{\omega}$, where $\bar{\omega}$ – normalized angular frequency, $\bar{\omega}=2\pi\frac{f}{f_d}$, $\bar{\omega}\in[0,\pi]$, f, f_d – respectively, linear frequency and sampling rate, in the numerator and denominator we will get complex transfer coefficient, and based on it PFR of low-pass filters (LPF) and high-pass filters (HPF).

Phase-frequency response in general form is described by expression

$$\varphi(\bar{\omega})=-\frac{\bar{\omega}}{2}+\mathrm{arctg}\left(\frac{b\sin\bar{\omega}}{1+b\cos\bar{\omega}}\right). \qquad (2)$$

From this expression follows, that PFR has linear component in the form of "$-\frac{\bar{\omega}}{2}$" and distorting linearity – second component, which depends only on the coefficient of the denominator b.

Let's define condition under which the second component is equal to the zero. It will be carried out at a frequency $\bar{\omega}=0$ and at the coefficient of denominator $b=0$. The first condition, is executable, but the second condition must be checked, as coefficient b depends on the cut-off frequency $\bar{\omega}_c$ and FRF ripple level ε [10]

$$b=-\left\{1-\frac{2\varepsilon^2\sin^2\left(\frac{\bar{\omega}_c}{2}\right)}{\varepsilon^2-\cos^2\left(\frac{\bar{\omega}_c}{2}\right)}\left(1-\frac{\cos\left(\frac{\bar{\omega}_c}{2}\right)}{\sin\left(\frac{\bar{\omega}_c}{2}\right)}\sqrt{\frac{1-\varepsilon^2}{\varepsilon^2}}\right)\right\}. \qquad (3)$$

Equating (3) to zero, we find that

$$\varepsilon=\cos\left(\frac{\bar{\omega}_c}{2}\right), \qquad (4)$$

i.e. if necessary to ensure PFR linearity of recursive LPF it is necessary to choose FRF ripple level at which cut-off frequency is determined , from the condition (4).

However, during frequency change of the filter it is necessary to ensure quasilinearity of PFR with and error Δ in some frequency range of adjustment. We will define frequency of exit outside an error Δ from a condition

$$-\frac{\bar{\omega}}{2}-\Delta\le\varphi(\bar{\omega})\le-\frac{\bar{\omega}}{2}+\Delta. \qquad (5)$$

Substituting (2) into (5) we will get a new condition

$$-\Delta \le \operatorname{arctg}\left(\frac{b\sin\bar{\omega}}{1+b\cos\bar{\omega}}\right) \le \Delta ,$$

which allows to define boundary frequency in the pass band $\bar{\omega}_l$ of PFR linearity in the range of $\pm\Delta$

$$\bar{\omega}_l = -2\operatorname{arctg}\left\{\frac{1}{\operatorname{tg}\Delta\left(1-\frac{1}{b}\right)}\left[1-\sqrt{1+(\operatorname{tg}\Delta)^2\left(1-\frac{1}{b}\right)}\right]\right\}.$$

In this case group delay in the pass band will change from the value of $\tau(0)=\frac{1-b}{2(1+b)}$ or taking into consideration (3) from $\tau(0)=\frac{1}{2}\frac{\sqrt{1-\varepsilon^2}}{\varepsilon}\frac{1}{\operatorname{tg}\frac{\bar{\omega}_c}{2}}$ to the value

$$\tau(\bar{\omega})=\frac{1-b^2}{2\left(1+b^2+2b\cos\bar{\omega}\right)} \text{ at } \bar{\omega}=\bar{\omega}_l .$$

For HPF phase response characteristic in general form is equal

$$\varphi(\bar{\omega})=\frac{\pi}{2}-\frac{\bar{\omega}}{2}+\operatorname{arctg}\left(\frac{b\sin\bar{\omega}}{1+b\cos\bar{\omega}}\right).$$

Linear PRC will take place by analogy with LPF on condition $b=0$. The dependence of coefficient b of HPF from cut-off frequency $\bar{\omega}_c$ and FRF ripple level ε is considered [1] and looks like

$$b=1-\frac{2\varepsilon^2\cos^2\left(\frac{\bar{\omega}_c}{2}\right)}{\varepsilon^2-\sin^2\left(\frac{\bar{\omega}_c}{2}\right)}\left(1-\frac{\sin\left(\frac{\bar{\omega}_c}{2}\right)}{\cos\left(\frac{\bar{\omega}_c}{2}\right)}\sqrt{\frac{1-\varepsilon^2}{\varepsilon^2}}\right). \quad (6)$$

Equating (6) to zero, we find also that

$$\varepsilon=\sin\left(\frac{\bar{\omega}_c}{2}\right), \quad (7)$$

i.e. if necessary to ensure PFR linearity of recursive HPF it is necessary to choose FRF ripple level at which cut-off frequency is determined , from the condition (7).

To ensure PFR quasilinearity with error Δ in some frequency range of adjustment by analogy with LPF it is necessary to consider the condition

$$-\Delta-\frac{\pi}{2} \le \operatorname{arctg}\left(\frac{b\sin\bar{\omega}}{1+b\cos\bar{\omega}}\right) \le \Delta-\frac{\pi}{2} ,$$

which allows to define boundary frequency in the pass band $\bar{\omega}_l$ of PFR linearity in the range of $\pm\Delta$

$$\bar{\omega}_l = 2\operatorname{arctg}\left\{\frac{1}{\operatorname{ctg}\Delta\left(1-\frac{1}{b}\right)}\left[1-\sqrt{1+(\operatorname{ctg}\Delta)^2\left(1-\frac{1}{b^2}\right)}\right]\right\}.$$

In this case group delay in the passband will change from the value of $\tau(\pi)=\frac{1-b}{2(1+b)}$ or taking into consideration (3) from $\tau(\pi)=\frac{1}{2}\frac{\sqrt{1-\varepsilon^2}}{\varepsilon}\frac{1}{\operatorname{tg}\frac{\bar{\omega}_c}{2}}$ to the value

$$\tau(\bar{\omega})=\frac{1-b^2}{2\left(1+b^2+2b\cos\bar{\omega}\right)} \text{ at } \bar{\omega}=\bar{\omega}_l .$$

It should be noted that the group delay in the pass band is equally determined for LPF and HPF.

It is possible to do next conclusions from the conducted analysis of PFR.

Phase-frequency response of first order digital filters, elliptic Chebyshev filter and inverse Chebysev filter with the same value of ripple in the passband and stopband coinside respectively, therefore in future will be considered only Chebyshev filter.

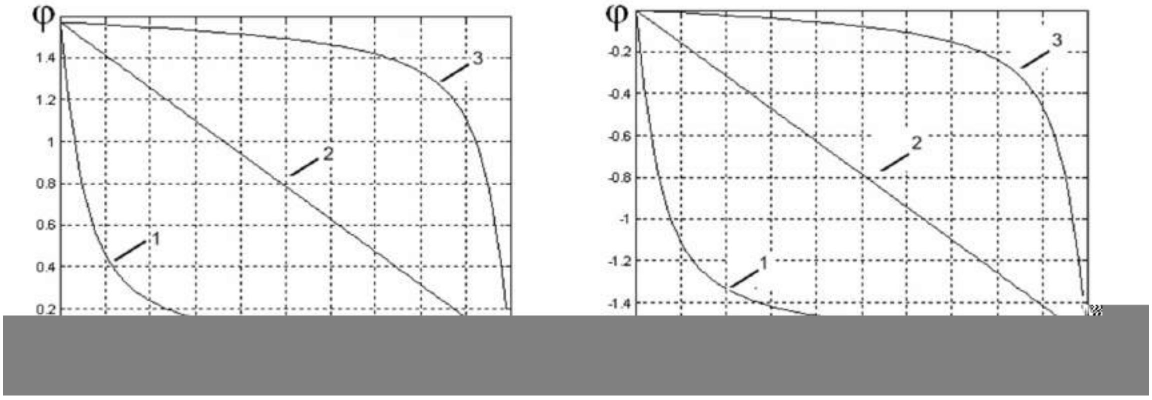
Phase-frequency response of Butterworth filter has the same form taking into account a sign both for LPF and for HPF. Also the same relationship retained for different values of the cut-off frequency.

It should be noted, that when cut-off frequency $\bar{\omega}_c=\frac{\pi}{2}$ — PFR is linear, in accordance with (4) and (7), Fig. 1.

Chebyshev high pass filter with ripple in the stopband $\varepsilon<|-3dB|$ has PFR similar to PFR of Butterworth filter, but with large nonlinear distortions toward cut-off low frequencies, Fig. 2, a and Fig. 3, a, and at $\varepsilon>|-3dB|$ — toward cut-off high frequencies, Fig. 3, в. At $\varepsilon=|-3dB|$ PFRs of Butterworth and Chebyshev filters are same, Fig. 1, and Fig. 3, б.

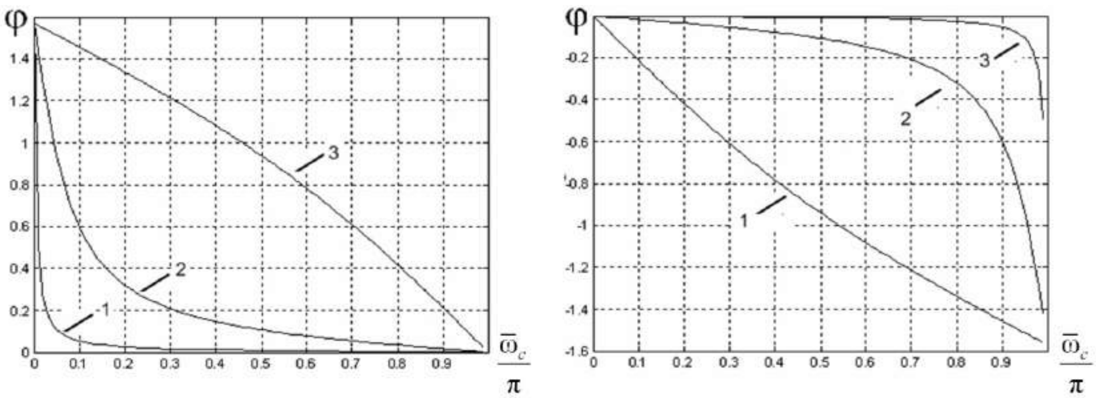
For LPF we see reverse pattern. Thus, when ripple in the pass band $\varepsilon < |-3dB|$ PFR is similar to PFR of Butterworth filter, but with large nonlinear distortions toward cut-off high frequencies Fig. 2, b, and at $\varepsilon > |-3dB|$ — toward cut-off low frequencies. At $\varepsilon = |-3dB|$ PFRs of Butterworth and Chebyshev filters also are same.

In all considered cases PFR in the passband of corresponding filter has segments close to linear. Dependence of phase from ripple level at given cut-off requency for Chebyshev first order HPF and LPF presented at three-dimensional graphs, Fig. 4. In Fig. 4 there are planes , at which PFR is linear, which allows to find areas of phase quasilinearity in passband for given ripple values.



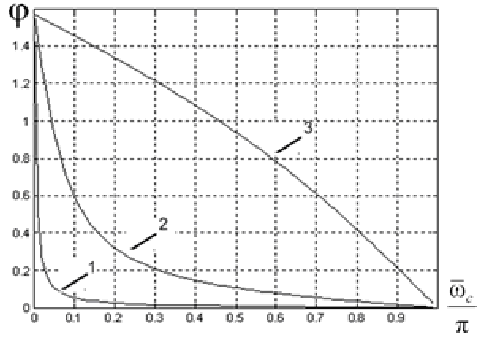
a b

Fig. 1. Phase-frequency response graphs for Butterworth filter:
a – HPF; b – LPF;
1 – $\bar{\omega}_c$ = 0,05; 2 – $\bar{\omega}_c$ = 0,5; 3 – $\bar{\omega}_c$ = 0,95

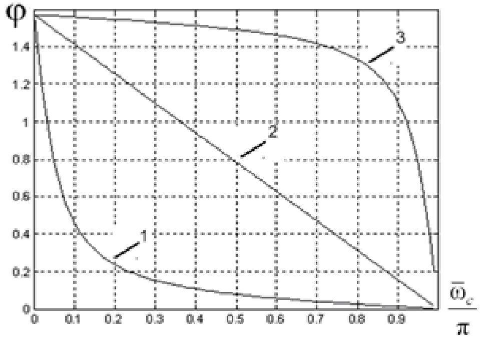


a b

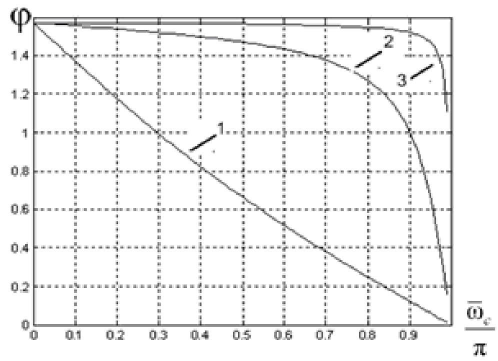
Fig. 2. Phase-frequency response graphs for Chebyshev filter with ripple level ε = –0,05 *dB*:
a – HPF; b – LPF;
1 – $\bar{\omega}_c$ = 0,05; 2 – $\bar{\omega}_c$ = 0,5; 3 – $\bar{\omega}_c$ = 0,95



a

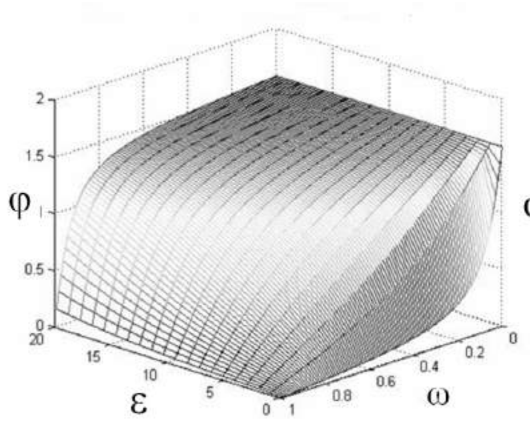


b

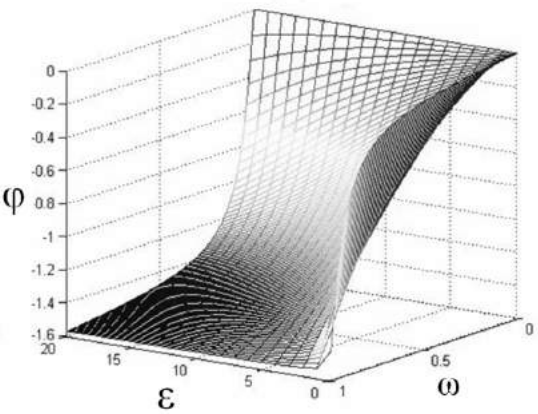


c

Fig. 3. Phase–frequency response graphs for Chebyshev HPF with ripple level:
- $\varepsilon = -0{,}05$ *dB* (a); - $\varepsilon = -3$ *dB* (b); - $\varepsilon = -20$ *dB* (c);
1 – $\overline{\omega}_c = 0{,}05$; 2 – $\overline{\omega}_c = 0{,}5$; 3 – $\overline{\omega}_c = 0{,}95$



a



b

Fig. 4. Graphs of dependence of phase from ripple level at given cut-off requency for Chebyshev first order HPF (a) and LPF (b)

Analysis of filters PRC showed that commonly used types of filters have a number of peculiarities:

1. PFR of first order filters is the same for ripple in the stopband at level $\varepsilon = -3dB$.

2. When you set the same values of ripple in the stopband Chebyshev filter and elliptical filter, elliptic Chebyshev filter and first order inverse Chebysev filter has the same PFRs.

3. For LPF and HPF we see reverse pattern, phase change when changing the cut-off frequency and corresponding ripple.

4. Group delay in the pass band is equally determined for LPF and HPF.

5. Depending on the magnitude of PFR linearity error Δ and denominator coefficient b quasilinearity limits of PFR are determined.

6. Taking into account listed above dependences, in PFR passband it is possible to take into account в phase distortions caused by filter in the further processing of signal.

The performed analysis allows to approach to designing of tract pre-filtering and processing with parameter changes of physical and chemical processes at the nuclear power station and during the medical and biological researches more consciously.

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Sytnikov Valery Stepanovich,
Doctor of Technical Sciences, professor, Head of the Department of Computer Systems of Odessa national polytechnic university,
mobile: +38-048-734-85-13
sitnvs@mail.ru

Sytnikov Tikhon Valeryevich,
Bachelor of Odessa national polytechnic university,
mobile: +38-063-525-11-74
zzzvisperzzz@yandex.ru

Tymokhin Dmitry Aleksandrovich,
Master of Odessa national polytechnic university,
mobile: +38-063-244-44-55
dimasgsm@mail.ru