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A SUGGESTED DEVELOPMENT OF THE SCALAR ESTIMATION ERROR (Δ) FOR THE LINEX LOSS FUNCTION TO ESTIMATE THE THREE PARAMETERS FOR BURR-XII FROM THE TYPE–II HYBRID CENSORED DATA

Ахмед Дияаб Ахмед, Дижла Ибрагим Махди. **Пропозиція щодо знаходження похибки скалярної оцінки функції втрат Лінекс для оцінки трьох параметрів Burr-XII на основі гібрідних цензурованих даних.** Гібридна схема цензури являє собою суміш типу I і II типів схем цензури. Знаходиться похибка скалярної оцінки Δ з функції втрат Лінекс і розглядається оцінка трьох параметрів Burr-X II на основі гібридних цензурованих даних II типу.Параметри оцінюються з допомогою наближення Ліндлі і Гіббс вибірки моделювання методом Монте Карло. Для порівняння використовувались різні методи і проаналізовано один набір даних для ілюстрації.
Ключові слова: Burr-XII розподіли, байєсівські оцінки, Лінекс функція втрат, гібридні цензуровані дані II типу.

Ахмед Дияаб Ахмед, Діжла Ібрагім Махді. **Предложение по нахождению погрешности скалярной оценки функции потерь Линекс для оценки трех параметров Burr-XII на основе гибридных цензурированных данных.** Гибридная схема цензуры представляет собой смесь типов I и II схем цензуры. Находится погрешность скалярной оценки Δ из функции потерь Линекс и рассмотривается оценка трех параметров Burr-X II на основе метода. Параметры оцениваются с помощью приближения Линдли и Гиббс выборки по методу Монте-Карло Для сравнения использовались различные методы и проанализирован один набор данных для иллюстрации.
Ключевые слова: Burr-XII распределения, байесовские оцени, Линекс функция потерь, гибридные цензурованные данные II типа.

Ahmed Dheyab Ahmed, Dejela Ibrahim Mahdi. **A Suggested development of the scalar estimation error (Δ) for the Linex Loss Function to estimate the three parameters for Burr-XII from the type–II hybrid censored data.** The hybrid censoring scheme is a mixture of type-I and type-II censoring schemes. In this paper we develop the scalar estimation error Δ from the linex loss function and we consider the estimation of the three parameters of Burr-X II based on type-II hybrid censored data. The parameters are estimated by the Lindley Approximation and Gibbs Sampling. Monte Carlo is simulation used to compare the different methods and we analyse one data set for illustrative purposes.
Keywords: Burr-XII distribution, Bayes estimators, Linex loss function, Type-II hybrid censoring

1. Introduction

The Burr-XII distribution is proposed by Burr [3] and it's starting to work in it for the last three decades. It's applied in reliability studies, failure time modeling and areas of quality control [8].

The three parameters Burr-XII probability density function (pdf) can be obtained by compounding a weibull probability density function with a gamma probability density function from the idea of Takahasi [11].

In this paper, we change the pdf of gamma distribution to obtaine the 3-parameters Burr-XII probability density function. That is

If x/θ ~ Weibull (θ,α) and θ ~ gamma (γ,λ) then the compound pdf, say $f(x\mid\alpha,\gamma,\lambda)$ is given by

$$f(x\mid\alpha,\gamma,\lambda)=\int_0^{\infty}\left[\theta\alpha x^{\alpha-1}e^{-\theta x^{\alpha}}\right]\left[\frac{\lambda^{\gamma}}{\Gamma(\gamma)}\theta^{\gamma-1}e^{-\lambda\theta}\right]d\theta, \quad (1)$$

$$f(x\mid\alpha,\gamma,\lambda)=\frac{\gamma\alpha}{\lambda}x^{\alpha-1}\left(1+\left(\frac{x^{\alpha}}{\lambda}\right)\right)^{-\gamma-1}, \qquad x\succ 0, \quad (2)$$

which is the 3-parameter Burr-XII (α,γ,λ) pdf, where

α, γ — shape parameters $\alpha\succ 0$, $\gamma\succ 0$

λ — scale parameters $\lambda\succ 0$

And the cumulative distribution function (cdf)

$$F(x\mid\alpha,\gamma,\lambda)=1-\left(1+\frac{x^{\alpha}}{\lambda}\right)^{-\gamma}, \qquad x\succ 0 \quad (3)$$

Type-I and type-II censoring schemes are the two most popular censoring schemes which are used in the reliability and life testing experiments Kundu and Howlader [7], Panahi and Asadi [9,10]. They discus the censoring schemes and others. The hybrid censoring scheme is a mixture of type-I and type-II censoring scheme. The type-II hybrid censoring scheme can be described as follows, put identical items n on test, and then terminate the experiment at the random time $T^*=\max\left(X_{R,n},T\right)$, where R and T are pre fixed numbers [4], has the advantage of guaranteeing that at least R failures are observed. Banerjee and Kundu [6], Panahi and Asadi [4] They used the type-II hybrid censoring schemes in Weibull and Burr-XII distribution respectively.

The first was introduced to hybrid censoring scheme by Epstein [5] under the assumption of exponential lifetime distribution of the experimental unit where he analyzed the data [1]. The first aim of this paper is to suggest development for the scalar estimation error (Δ) from the linex loss function, and the second aim is to estimate the parameters by using the Approximation Bayes estimators and Gibbs sampling.

2. Bayesian Analysis

The Bayes estimation of the unknown parameters (ϑ) under the linex loss function is given by

$$\therefore\ \hat{\theta}_{Linex}=-\frac{1}{C}Ln\,E_{\theta}\left(e^{-C\theta}\right), \quad (4)$$

where, the Linex Loss function is given by

$$L_{Linex}(\Delta)=e^{C\Delta}-C\Delta-1, \qquad C\neq 0, \quad (5)$$

where

$\Delta=\left(\hat{\theta}-\theta\right)$ is a scalar estimation error using $\hat{\vartheta}$ to estimate ϑ, here we found Lindley estimator and Gibbs sampling, (see appendix B to compute Gibbs sampling).

We suppose that a scalar estimation error can be writtin as follows:

$$\Delta=\left(\hat{\theta}^{m}-\theta^{m}\right),\ m=2,3,4. \quad (6)$$

By using Lindley approximation of the unknown parameters under the change as in equation (6) then, the Bayes estimation became

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$$\therefore\ \hat{\theta}_{Linex} = \sqrt[m]{-\frac{1}{C} Ln\, E_{\theta}\left(e^{-C\overset{m}{\theta}}\right)}\ . \tag{7}$$

We assume α,γ and λ have the prior distribution (gamma distribution)

$$\Pi_1(\alpha) \propto \alpha^{a_1-1} e^{-a_2\alpha}, \qquad \alpha \succ 0\ ,\ (\alpha_1 a_2) \succ 0, \tag{8}$$

$$\Pi_2(\gamma) \propto \gamma^{a_3-1} e^{-a_4\gamma}, \qquad \gamma \succ 0,\ (\alpha_3, a_4) \succ 0, \tag{9}$$

$$\Pi_3(\lambda) \propto \lambda^{a_5-1} e^{-a_6\lambda} \qquad \lambda \succ 0,\ (\alpha_5, a_6) \succ 0. \tag{10}$$

And the joint prior density function for the parameters α,γ,λ is given by

$$\Pi(\alpha,\gamma,\lambda) = \Pi_1(\alpha)\Pi_2(\gamma)\Pi_3(\lambda). \tag{11}$$

Based on (11) the joint posterior density of α,γ and λ is

$$\Pi(\alpha,\gamma,\lambda \mid x) = \frac{\Pi(x \mid \alpha,\gamma,\lambda)}{\int_0^\infty\int_0^\infty\int_0^\infty \Pi(x \mid \alpha,\gamma,\lambda) d\alpha d\gamma d\lambda}. \tag{12}$$

Therefore, the Bayes estimate of any function of α,γ and λ say $g(\alpha,\gamma,\lambda)$, under the change of Linex Loss function is given by

$$\hat{g}_{Linex}(\alpha,\gamma,\lambda) = \sqrt[m]{\left(-\frac{1}{C} Ln\, E_{\alpha,\gamma,\lambda|x}\left[g(\alpha,\gamma,\lambda)\right]\right)}, \tag{13}$$

where

$$E_{\alpha,\gamma,\lambda|x}\left[g(\alpha,\gamma,\lambda)\right] = \frac{\int_0^\infty\int_0^\infty\int_0^\infty \left[g(\alpha,\gamma,\lambda)\right]\Pi(x \mid \alpha,\gamma,\lambda) d\alpha d\gamma d\lambda}{\int_0^\infty\int_0^\infty\int_0^\infty \Pi(x \mid \alpha,\gamma,\lambda) d\alpha d\gamma d\lambda}. \tag{14}$$

It is not possible to compute (14) analytically; therefore, we used Lindley Approximation to compute Bayes estimators, that his Approximation formula as follows:

$$E\left(u(\underline{\theta})/x\right) = \frac{\int_\Omega u(\theta).e^{L(\theta)+\rho(\theta)} d\theta}{\int_\Omega e^{L(\theta)+\rho(\theta)} d\theta}, \tag{15}$$

$$\begin{aligned} &\cong u(\hat{\theta}) + 0,5\sum_{i=1}^{\varepsilon}\sum_{j=1}^{\varepsilon}\left[\left.\frac{\partial^2 u(\theta)}{\partial\theta_i \partial\theta_j}\right|_{\theta=\hat{\theta}} + \left\{2\left.\frac{\partial u(\theta)}{\partial\theta_i}\right|_{\theta=\hat{\theta}}\right\}\left\{\left.\frac{\partial\rho(\theta)}{\partial\theta_j}\right|_{\theta=\hat{\theta}}\right\}\right]\hat{\sigma}_{ij} + \\ &+\left\{0,5\sum_{i=1}^{\varepsilon}\sum_{j=1}^{\varepsilon}\sum_{k=1}^{\varepsilon}\sum_{l=1}^{\varepsilon}\left.\frac{\partial^3 L(\theta)}{\partial\theta_i\partial\theta_j\partial\theta_l}\right|_{\theta=\hat{\theta}}\right\}\bullet\left\{\left.\frac{\partial u(\theta)}{\partial\theta_k}\right|_{\theta=\hat{\theta}}\right\}\hat{\sigma}_{ij}\hat{\sigma}_{kl}, \end{aligned} \tag{16}$$

where

ε the number of parameters, $\hat{\theta}$ is maximum Likelihood estimator, $\Pi(\theta)$ the prior distribution and

$$L_{ijl} = \left.\frac{\partial^3 L(\theta)}{\partial\theta_i\partial\theta_j\partial\theta_l}\right|_{\theta=\hat{\theta}},\quad \sigma_{ij} = -\left(\left.\frac{\partial L^2(\theta)}{\partial\theta_i\partial\theta_j}\right|_{\theta=\hat{\theta}}\right)^{-1},$$

where

$i, j, k,$ l =1, 2, 3

For the detailed derivations, see the Appendix *A*.

Thus, based on (7) and (16), the approximate Bayes estimates of α, γ and λ when we changed Linex Loss function are respectively

$$\hat{\alpha} = -\frac{1}{C} Log \left\{ \begin{array}{l} e^{-C\hat{\alpha}^{m}} + \frac{1}{2}\left[\left(C^2 m^2 \hat{\alpha}^{2(m-1)} e^{-C\hat{\alpha}^{m}} - Cm(m-1)\hat{\alpha}^{m-2} e^{-C\hat{\alpha}^{m}}\right)\sigma_{11}\right] - \\ Cm\hat{\alpha}^{m-1} e^{-C\hat{\alpha}^{m}} \left(\begin{array}{l} \sigma_{11}\left(\frac{a_1 - 1}{\hat{\alpha}} - a_2\right) + \sigma_{12}\left(\frac{a_3 - 1}{\hat{\gamma}} - a_4\right) \\ + \sigma_{13}\left(\frac{a_5 - 1}{\hat{\lambda}} - a_6\right) \end{array} \right) - \\ \frac{1}{2} Cm \hat{\alpha}^{m-1} e^{-C\hat{\alpha}^{m}} \left[\begin{array}{l} L_{111}\sigma_{11}\sigma_{12} + L_{112}\sigma_{11}\sigma_{21} + L_{113}\sigma_{11}\sigma_{31} + L_{121}\sigma_{12}\sigma_{11} + \\ L_{123}\sigma_{12}\sigma_{31} + L_{131}\sigma_{13}\sigma_{11} + L_{132}\sigma_{13}\sigma_{21} + L_{133}\sigma_{13}\sigma_{31} + \\ L_{211}\sigma_{21}\sigma_{11} + L_{213}\sigma_{21}\sigma_{31} + L_{222}\sigma_{22}\sigma_{21} + L_{231}\sigma_{23}\sigma_{11} + \\ L_{233}\sigma_{23}\sigma_{31} + L_{311}\sigma_{31}\sigma_{11} + L_{312}\sigma_{31}\sigma_{21} + L_{313}\sigma_{31}\sigma_{31} + \\ L_{321}\sigma_{32}\sigma_{11} + L_{323}\sigma_{32}\sigma_{31} + L_{331}\sigma_{33}\sigma_{11} + L_{332}\sigma_{33}\sigma_{21} + \\ L_{333}\sigma_{33}\sigma_{31} \end{array} \right] \end{array} \right\} = A1,$$

$$\hat{\alpha} = \sqrt[m]{A1} \qquad (17)$$

$$\hat{\gamma} = -\frac{1}{C} Log \left\{ \begin{array}{l} e^{-C\hat{\gamma}^{m}} + \frac{1}{2}\left[\left(C^2 m^2 \hat{\gamma}^{2(m-1)} e^{-C\hat{\gamma}^{m}} - Cm(m-1)\hat{\gamma}^{m-2} e^{-C\hat{\gamma}^{m}}\right)\sigma_{22}\right] - \\ Cm\hat{\gamma}^{m-1} e^{-C\hat{\gamma}^{m}} \left(\begin{array}{l} \sigma_{21}\left(\frac{a_1 - 1}{\hat{\alpha}} - a_2\right) + \sigma_{22}\left(\frac{a_3 - 1}{\hat{\gamma}} - a_4\right) \\ + \sigma_{23}\left(\frac{a_5 - 1}{\hat{\lambda}} - a_6\right) \end{array} \right) - \\ \frac{1}{2} Cm \hat{\gamma}^{m-1} e^{-C\hat{\gamma}^{m}} \left[\begin{array}{l} L_{111}\sigma_{11}\sigma_{12} + L_{112}\sigma_{11}\sigma_{22} + L_{113}\sigma_{11}\sigma_{32} + L_{121}\sigma_{12}\sigma_{12} + \\ L_{123}\sigma_{12}\sigma_{32} + L_{131}\sigma_{12}\sigma_{12} + L_{132}\sigma_{13}\sigma_{22} + L_{133}\sigma_{13}\sigma_{32} + \\ L_{211}\sigma_{21}\sigma_{12} + L_{213}\sigma_{21}\sigma_{32} + L_{222}\sigma_{22}\sigma_{21} + L_{231}\sigma_{22}\sigma_{12} + \\ L_{233}\sigma_{23}\sigma_{32} + L_{311}\sigma_{31}\sigma_{12} + L_{312}\sigma_{31}\sigma_{22} + L_{313}\sigma_{31}\sigma_{32} + \\ L_{321}\sigma_{32}\sigma_{12} + L_{323}\sigma_{32}\sigma_{32} + L_{331}\sigma_{32}\sigma_{12} + L_{332}\sigma_{33}\sigma_{22} + \\ L_{333}\sigma_{33}\sigma_{32} \end{array} \right] \end{array} \right\} = B1,$$

$$\hat{\gamma} = \sqrt[m]{B1}, \qquad (18)$$

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$$\hat{\lambda} = -\frac{1}{C} Log \left\{ \begin{array}{l} e^{-C\hat{\lambda}^{m}} + \frac{1}{2}\left[\left(C^2 m^2 \hat{\lambda}^{2(m-1)} e^{-C\hat{\lambda}^{m}} - Cm(m-1)\hat{\lambda}^{m-2} e^{-C\hat{\lambda}^{m}} \right) \sigma_{33} \right] - \\ Cm \hat{\lambda}^{m-1} e^{-C\hat{\lambda}^{m}} \left(\begin{array}{l} \sigma_{31}\left(\frac{a_1 - 1}{\hat{\alpha}} - a_2 \right) + \sigma_{32}\left(\frac{a_{31} - 1}{\hat{\gamma}} - a_4 \right) \\ + \sigma_{33}\left(\frac{a_5 - 1}{\hat{\lambda}} - a_6 \right) \end{array} \right) - \\ \frac{1}{2} Cm \hat{\lambda}^{m-1} e^{-C\hat{\lambda}^{m}} \left[\begin{array}{l} L_{111}\sigma_{11}\sigma_{13} + L_{112}\sigma_{11}\sigma_{23} + L_{113}\sigma_{11}\sigma_{33} + L_{121}\sigma_{12}\sigma_{13} + \\ L_{123}\sigma_{12}\sigma_{33} + L_{131}\sigma_{13}\sigma_{13} + L_{132}\sigma_{13}\sigma_{23} + L_{133}\sigma_{13}\sigma_{33} + \\ L_{211}\sigma_{21}\sigma_{31} + L_{213}\sigma_{21}\sigma_{33} + L_{222}\sigma_{22}\sigma_{23} + L_{231}\sigma_{23}\sigma_{13} + \\ L_{233}\sigma_{23}\sigma_{33} + L_{311}\sigma_{31}\sigma_{13} + L_{312}\sigma_{31}\sigma_{23} + L_{313}\sigma_{31}\sigma_{33} + \\ L_{321}\sigma_{32}\sigma_{13} + L_{323}\sigma_{32}\sigma_{33} + L_{331}\sigma_{33}\sigma_{13} + L_{332}\sigma_{33}\sigma_{23} + \\ L_{333}\sigma_{33}\sigma_{33} \end{array} \right] \end{array} \right\} = C1.$$

$$\hat{\lambda} = \sqrt[m]{C1} \tag{19}$$

3. Simulation Results

In this section we carry out a simulation study to compare the performances of Bayesian procedure in different (m), in terms of the mean squared error (MSEs). The simulation is carried out for different choices of n, *R* and *T* values. We have taken all cases, when $\alpha =$ 2б5 , $\gamma = 1$ and $\lambda = 3$ and we have assumed that α, γ and λ have respectively $Gamma(a_1, a_2)$, $Gamma(a_3, a_4)$ and $Gamma(a_5, a_6)$ priors, where $a_1 = a_3 = a_5 = 1{,}5$ and $a_2 = a_4 = a_6 = 2$. We replicate the process 100 times. The results are summarized in Table 1 and Table 2.

4. Simulation Comparisons

Simulation comparisons of various (*m*) is made when *n*=10, 20, 30, 40, 50 with hybrid type-II censored data, From Table 1 and Table 2, above, it may be observed that the Bayes estimates when (*m*=2) are, generally, better than the Bayes estimates when (*m*=1, 3, 4 and 5) and Gibbs sampling (because it gives the smaller MSEs),

5. Real Life Data

In this section we analyse real life data set to estimate the three parameters for Burr-XII distribution by using Lindleys approximation when (*m*=2) and Gibbs sampling, The data which represent the Single thread strength and tested under gauge lengths of 50 cm, with sample size *n*=26 are: 3,85, 3,85, 3,9, 3,9, 3,95, 4, 4, 4,05, 4,1, 4,15, 4,2, 4,25, 4,25, 4,3, 4,3, 4,35, 4,35, 4,35, 4,4, 4,4, 4,4, 4,45, 4,5, 4,7, 4,8, 5.

Consider the following two sampling schemes (Note: we subtract 3,75 from the observations):

Scheme 1 is *R* = 13 and *T* = 1

And Scheme 2 is *R* = 8 and *T* = 0,7

The data are coming from the Burr-XII distribution by using statistical programming (Easy Fit 5,2 Professional), Table 3 shows the estimators of the three parameters of the Burr-XII distribution,

Table 1

The Mean Squared Errors (MSEs) of Lindleys approximation and Gibbs sampling for (T=1 and c=1)

n	R	Parameter	Lindley					Gibbs
			m_1	m_2	m_3	m_4	m_5	
10	3	α	8,5062e–006	8,1317e–006	1,7348e–005	3,5905e–005	4,9560e–005	5,0808e–001
		γ	1,2394e–006	9,2811e–006	3,2620e–005	8,0398e–005	1,2535e–004	5,0348e–001
		λ	2,9178e–006	2,4101e–006	1,7092e–005	3,4272e–005	4,7534e–005	4,2395e–001
	8	α	4,9420e–005	2,1535e–006	3,6413e–006	1,9951e–005	1,3762e–005	5,0576e–001
		γ	7,7108e–007	6,0846e–006	3,2062e–005	7,1428e–005	1,0541e–004	5,0405e–001
		λ	1,1746e–005	1,9755e–006	2,0327e–005	4,9056e–005	4,6490e–005	4,3990e–001
20	8	α	1,0154e–005	2,1395e–006	1,5619e–005	1,4847e–005	3,2752e–005	4,6453e–001
		γ	4,5250e–006	4,4416e–006	2,9537e–005	4,5308e–005	6,1943e–005	4,4215e–001
		λ	3,5979e–006	5,5249e–006	1,3430e–005	1,6581e–005	2,6640e–005	4,6422e–001
	12	α	4,5552e–005	1,0947e–006	1,9918e–005	2,3587e–005	3,3264e–005	4,6033e–001
		γ	1,4469e–006	1,1860e–005	2,6461e–005	4,0762e–005	6,7555e–005	4,5145e–001
		λ	4,9846e–006	1,4029e–006	7,5479e–006	2,6868e–005	2,9016e–005	4,4321e–001
30	15	α	1,3071e–005	2,9287e–006	1,1974e–005	1,9677e–005	2,5873e–005	4,7262e–001
		γ	1,0618e–006	5,7409e–006	1,7369e–005	3,1001e–005	5,2682e–005	4,5990e–001
		λ	1,3382e–005	3,8308–006	7,7240e–006	2,4120e–005	3,3233e–005	4,5478e–001
	22	α	3,0827e–005	1,5610e–006	7,0052e–006	1,8912e–005	1,4294e–005	4,8490e–001
		γ	1,7146e–006	1,2198e–005	1,4309e–005	2,3826e–005	4,6430e–005	4,8474e–001
		λ	1,5090e–005	5,2556e–006	4,8638e–006	1,7628e–005	2,6052e–005	4,6946e–001
40	28	α	1,2571e–005	3,9381e–006	1,4059e–005	1,0585e–005	1,7491e–005	4,5867e–001
		γ	9,5603e–006	4,9448e–006	1,8454e–005	8,1060e–005	7,2501e–005	4,5213e–001
		λ	2,0402e–005	5,3758e–006	1,2125e–005	1,9041e–005	3,1496e–005	4,5371e–001
	35	α	3,1035–005	5,5208e–006	9,6089e–006	1,4195e–005	1,3835e–005	4,6451e–001
		γ	7,3801e–006	1,2333e–005	1,9973e–005	2,7043e–005	3,3937e–005	4,6148e–001
		λ	1,4414e–005	6,9652e–006	7,9446e–006	1,8019e–005	2,7784e–005	4,7808e–001
50	36	α	1,4338e–005	5,8246e–006	5,2029e–006	1,0154e–005	2,1987e–005	4,0774e–001
		γ	1,0500e–005	1,3291e–005	1,4758e–005	2,0121e–005	6,0347e–005	4,0645e–001
		λ	1,1714e–005	5,4213e–006	8,2538e–006	1,0765e–005	1,9109e–005	4,9044e–001
	45	α	2,4025e–005	1,9830e–006	5,6542e–006	1,2374e–005	2,6637e–005	4,3884e–001
		γ	8,1902e–006	6,5045e–006	2,2107e–005	3,4215e–005	5,2571e–005	4,3759e–001
		λ	2,8147e–005	6,4772e–006	1,6752e–005	1,7502e–005	1,7350e–005	5,2264e–001

Table 2

The Mean Squared Errors (MSEs) of Lindleys approximation and Gibbs sampling for (T=1,5 and c=1)

n	R	Parameter	Lindley					Gibbs
			m_1	m_2	m_3	m_4	m_5	
10	3	α	1,7016e–005	4,3781e–006	1,8444e–005	1,8782e–005	5,5964e–005	5,0562e–001
		γ	6,8610e–006	1,4667e–005	3,6799e–005	5,0716e–005	1,0592e–004	4,9974e–001
		λ	4,7636e–006	4,3353e–006	1,8257e–005	3,0826e–005	5,4897e–005	4,2499e–001
	8	α	5,2317e–005	1,3796e–006	4,2510e–006	8,8464e–006	1,9003e–005	5,0392e–001
		γ	7,9655e–007	1,2015e–005	2,9222e–005	4,1539e–005	8,3324e–005	5,0876e–001
		λ	3,2351e–006	1,6437e–006	1,8937e–005	2,5355e–005	4,2842e–005	4,4707e–001
20	8	α	2,4686e–005	5,8849e–006	1,2764e–005	1,7964e–005	2,7234e–005	4,6624e–001
		γ	1,1189e–006	9,3248e–006	2,7362e–005	3,1307e–005	6,7505e–005	4,4990e–001
		λ	4,5242e–006	3,2301e–006	1,2776e–005	2,6553e–005	2,4414e–005	4,5969e–001
	12	α	1,9448e–005	2,7662e–006	1,0856e–005	1,8061e–005	2,2785e–005	4,5970e–001
		γ	7,8155e–007	4,6293e–006	3,3294e–005	3,4746e–005	4,9186e–005	4,3919e–001
		λ	9,4850e–006	8,1835e–006	6,5291e–006	1,2743e–005	2,0806e–005	4,2305e–001

n	*R*	Parameter	Lindley					Gibbs
			m_1	m_2	m_3	m_4	m_5	
30	15	α	1,6895e–005	7,2331e–006	1,3092e–005	1,2783e–005	2,5310e–005	4,7140e–001
		γ	2,5623e–006	7,3747e–006	1,3651e–005	4,2656e–005	7,050e–005	4,6336e–001
		λ	4,8187e–006	5,4615e–006	1,0594e–005	1,8453e–005	1,6119e–005	4,6201e–001
	22	α	3,7356e–005	6,0398e–006	1,1116e–005	2,2828e–005	1,5984e–005	4,8490e–001
		γ	4,8153e–006	1,4543e–005	1,5793e–005	4,3523e–005	2,9351e–005	4,7621e–001
		λ	1,6927e–005	8,2553e–006	3,5411e–006	2,1342e–005	1,4685e–005	4,5448e–001
40	28	α	1,6380e–005	4,7521e–006	7,8047e–006	1,3098e–005	1,2207e–005	4,5768e–001
		γ	3,0329e–006	9,8769e–006	2,1299e–005	6,7740e–005	5,3235e–005	4,5180e–001
		λ	1,2333e–005	8,2026e–006	4,8088e–006	1,6645e–005	2,2930e–005	4,5533e–001
	35	α	4,5031e–005	5,2617e–006	1,2534e–005	1,3693e–005	1,7866e–005	4,6308e–001
		γ	9,2455e–006	1,8007e–005	2,1736e–005	3,4829e–005	3,4765e–005	4,5876e–001
		λ	2,3170e–005	8,2082e–006	1,2559e–005	2,5831e–005	3,5786e–005	4,7648e–001
50	36	α	1,8499e–005	7,3096e–006	2,5208e–006	1,4239e–005	1,4719e–005	4,1141e–001
		γ	5,3093e–006	5,1008e–006	1,2691e–005	3,5267e–005	6,4552e–005	4,0795e–001
		λ	8,7650e–006	7,4356e–006	6,2970e–006	1,0131e–005	1,1219e–005	5,0677e–001
	45	α	4,9096e–005	3,2629e–006	1,0738e–005	1,3971e–005	1,6703e–005	4,4065e–001
		γ	8,2922e–006	9,3145e–006	1,9945e–005	2,1512e–005	3,6047e–005	4,3939e–001
		λ	1,3161e–005	9,2604e–006	1,3010e–005	1,4272e–005	2,8185e–005	5,0248e–001

Table 3

The estimate of the parameters of Lindleys approximation and Gibbs sampling for
($\alpha = 0{,}5$, $\gamma = 5$, $\lambda = 4$ and c=1)

R	*T*	Estimate	Lindley	Gibbs
8	0,7	$\hat{\alpha}$	4,8300e–001	3,8663e+000
		$\hat{\gamma}$	6,5775e–001	3,8479e+000
		$\hat{\lambda}$	1,8216e+000	3,8633e+000
13	1	$\hat{\alpha}$	5,0106e–001	4,5580e+000
		$\hat{\gamma}$	6,9251e–001	4,5481e+000
		$\hat{\lambda}$	1,9932e+000	4,5568e+000

6. Conclusions

In this paper we have considered the Bayesian procedure for the unknown parameters of the Burr-XII distribution when the data are type-II hybrid censored, It is observed that the Bayes estimators of the unknown parameters can not be obtained in explicit forms, We compare the performance of the different methods by Monte Carlo simulation and it is observed that the performances are quite satisfactory, We show that the Lindley approximation when (*m*=2) is better than Lindley approximation when (*m*=1, 3, 4, 5) and Gibbs sampling,

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